The number of elements that square to the identity in the symmetric group $S_n$ is determined by a well-known recursion. We study a generalization of this question: for which values of $k$ are there exactly $k$ elements such that $x^k$ is the identity? Given any $n$, we determine the greatest $k < n!$ and the least $k > 1$ that satisfy this condition. (Received September 20, 2016)