M urges Xu* (xumurong@math.wvu.edu), 7205 University Commons Dr., Morgantown, WV 26505, and Janet Anderson, Suohai Fan, Hong-Jian Lai and Xiaoxia Lin. An extremal problem in digraph connectivity.

A digraph $D$ is strong if for any pair of vertices $u, v \in V(D)$, $D$ always contains a $(u, v)$-dipath. The strong arc connectivity of a digraph $D$, denoted by $\lambda(D)$, is the minimum number of arcs whose removal results in a non-strong digraph. If we just count the number of arcs in a digraph, can we predict that $D$ contains a subdigraph with high strong arc connectivity? We define $\bar{\lambda}(D) = \max\{\lambda(H) : H \subseteq D\}$. Given an integer $k > 0$, a strict digraph $D$ is $k$-maximal if $\bar{\lambda}(D) \leq k$ but adding any arc which is not in $D$ will surely create a subdigraph with strong arc connectivity at least $k+1$. Mader [Math. Ann. 1971] and Lai [JGT 1990] studied the extremal size of undirected $k$-maximal graphs. In this project, we determine that if $D$ is a $k$-maximal digraph on $n > k$ vertices, then

$$
\binom{n}{2} + (n-1)k + \lfloor \frac{n}{k+2} \rfloor \left(1 + 2k - \binom{k+2}{2} \right) \leq |A(D)| \leq k(2n - k - 1) + \binom{n-k}{2}.
$$

Consequently, if $|A(D)| > k(2n - k - 1) + \binom{n-k}{2}$, then $D$ must have a nontrivial subdigraph $H$ such that the strong arc connectivity of $H$ is at least $k+1$. (Received September 12, 2016)