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Bobby C. Shen* (runbobby@mit.edu). *The Parametric Frobenius Problem.*

The Frobenius number of relatively prime positive integers a_1, \dots, a_n , which we denote by $F(a_1, \dots, a_n)$, is the largest integer that is not a nonnegative integer combination of the a_i . We extend this to all positive integers: let $F(a_1, \dots, a_n)$ be the largest multiple of $\gcd(a_1, \dots, a_n)$ which is not a nonnegative integer combination of the a_i . We consider a parametric version of the Frobenius problem. Let P_1, \dots, P_n be polynomials from \mathbb{Z} to \mathbb{Z} with positive leading coefficients, and let $Q(t) = F(P_1(t), \dots, P_n(t))$, which is defined for sufficiently large integers t . We prove that this function is eventually quasi-polynomial; Q is eventually quasi-polynomial if there exists a positive integer d and polynomials R_0, \dots, R_{d-1} such that for such that for sufficiently large t , $Q(t) = R_{t \pmod{d}}(t)$. We do so by forming a parametric integer linear program whose optimum value at t equals $Q(t)$ for sufficiently large t . Using these ideas, one can show that if $n > 1$ and p, m are positive integers, then the p^{th} largest multiple of $\gcd(a_1, \dots, a_n)$ which is a nonnegative integer combination of the a_i in less than m ways is eventually quasi-polynomial in t (not in the presentation). (Received September 22, 2015)