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Rediet Abebe* (rta36@cornell.edu). *A Bound for the Laplacian Spectra of Simplicial Complexes.*

We present a generalization of Brouwer's conjectural family of inequalities for the Laplacian spectrum of graphs to the case of abstract simplicial complexes. This conjecture states, given a graph G with e edges and Laplacian eigenvalues $(\lambda_1 \geq \lambda_2 \geq \dots \lambda_{n-1} \geq \lambda_n = 0)$,

$$\sum_{i=1}^t \lambda_i \leq e + \binom{t+1}{2}, \forall t \in [n].$$

We generalize to the case of abstract simplicial complexes (of any dimension $k - 1$) with Laplacian eigenvalues $\lambda(S) = (\lambda_1 \geq \lambda_2 \geq \dots)$ to the inequality,

$$\sum_{i=1}^t \lambda_i \leq (k - 1)f_{k-1}(S) + \binom{t + k - 1}{k},$$

where f_{k-1} is the number of facets.

We prove this family of inequalities for shifted simplicial complexes and give tighter bounds (linear in the dimensions of the complexes) for simplicial trees. We show that the conjecture holds for the t^{th} partial sum for all simplicial complexes with dimension at least t and matching number greater than t as well as resolve the case for the first, second, and last partial sum completely. We also expand on a known proof for graphs to show that the conjecture holds with equality for threshold graphs when t is the number of cone vertices. (Received September 22, 2015)