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Daniel Alan Spielman* (spielman@cs.yale.edu). *Graphs, Vectors, and Matrices*.

I will explain how we use linear algebra to understand graphs and how recently developed ideas in graph theory have inspired progress in linear algebra.

Graphs can take many forms, from social networks to road networks, and from protein interaction networks to scientific meshes. One of the most effective ways to understand the large-scale structure of a graph is to study algebraic properties of matrices we associate with it. I will give examples of what we can learn from the Laplacian matrix of a graph.

We will use the graph Laplacian to define a notion of what it means for one graph to approximate another, and we will see that every graph can be well-approximated by a graph having few edges. For example, the best sparse approximations of complete graphs are provided by the famous Ramanujan graphs. As the Laplacian matrix of a graph is a sum of outer products of vectors, one for each edge, the problem of sparsifying a general graph can be recast as a problem of approximating a collection of vectors by a small subset of those vectors. The resulting problem appears similar to the problem of Kadison and Singer in Operator Theory. We will sketch how research on the sparsification of graphs led to its solution. (Received September 14, 2015)