Can we generalize the limit-definition of the derivative? Preliminary report.

Let $f : D \to \mathbb{R}$ and $x \in (a, b) \subset D$. Let $g : E \to \mathbb{R}$ such that $x + \epsilon g(\xi) \in D$ for $\xi \in E$ and some $\epsilon > 0$. We consider the derivative given by

$$D_{g(\xi)}(f)(x) = \lim_{\epsilon \to 0} \frac{f(x + \epsilon g(\xi)) - f(x)}{\epsilon}$$

relative to the function $g$ and prove that it obeys the familiar properties such as product rule, quotient rule, power rule, chain rule, Mean Value Theorems, Rolle’s Theorem and several other properties that a standard derivative satisfies. It can be seen that the function $g$ acts as a catalyst and controls the convergence rate. We also discuss the relation between this derivative and two other existing derivatives, namely the Fréchet and Gâteaux derivatives. According to the literature, it is interesting to note that the case of $g(x) = x^{1-\alpha}$, is now known as the Katugampola fractional derivative. Finally, we also point out that the definition can easily be extended to include the complex-valued functions. (Received September 23, 2015)