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**Xiaoyue Cui\*** (cuix.wsu@gmail.com), 727 W Martin Luther King Drive, Apt. 514W,  
Cincinnati, OH 45220, and **Guozhen Lu**. *New Characterizations of High Order Sobolev Space on  
Euclidean Spaces.*

The main purpose of this paper is to study some new characterizations of the high order Sobolev spaces  $W^{m,p}(\mathbb{R}^N)$ .

Actually, we will present here two types of characterizations: by m-th differences and by m-Taylor reminder. Therefore, two of our main results are as follows: If  $f \in L^p(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ , then

(1)  $f \in W^{m,p}(\mathbb{R}^N)$  if and only if

$$\sup_{0 < \delta < 1} \int_{\mathbb{R}^N} \frac{\delta^p}{|x - y|^{N+mp}} dx dy < \infty.$$
$$|\Delta^{(m)} f(x,y)| > \delta$$

(2)  $f \in W^{m,p}(\mathbb{R}^N)$  if and only if

$$\sup_{0 < \delta < 1} \int_{\mathbb{R}^N} \int_{\mathbb{R}^N} \frac{\delta^p}{|x - y|^{N+mp}} dx dy < \infty, f \in W^{m-1,p}(\mathbb{R}^N)$$
$$|R^{m-1} f(x,y)| > \delta$$

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