We study finite element approximations of the stochastic Allen-Cahn equation with gradient-type multiplicative noises that are white in time and correlated in space. The sharp interface limit as the parameter $\epsilon \to 0$ of the stochastic equation formally approximates a stochastic mean curvature flow which is described by a stochastically perturbed geometric law of the deterministic mean curvature flow. Two fully discrete finite element methods which are based on different time-stepping strategies for the nonlinear term are proposed. Strong convergence with sharp rates for both fully discrete finite element methods is proved with a crucial help of the Hölder continuity in time with respect to the spatial $L^2$-norm and $H^1$-seminorm for the strong solution of the stochastic Allen-Cahn equation. It also relies on the fact that high moments of the strong solution are bounded in various spatial and temporal norms. Numerical experiments are provided to gauge the performance of the proposed fully discrete finite element methods and to study the interplay of the geometric evolution and gradient-type noises. (Received September 21, 2015)