

1135-15-2824

Sima Ahsani* (sza0043@auburn.edu), 221 Parker Hall, Auburn University, Auburn, AL 36849.

Inequalities Related to Geometric Mean of Matrices. Preliminary report.

An $n \times n$ complex matrix A is called positive definite if it is hermitian and all its eigenvalues are positive. The set of all positive definite matrices denoted by \mathbb{P}_n and form a Riemannian manifold. The geodesic connecting $A, B \in \mathbb{P}_n$, is $\gamma(t) = A^{\frac{1}{2}}(A^{-\frac{1}{2}}BA^{-\frac{1}{2}})^t A^{\frac{1}{2}}, t \in [0, 1]$, and the midpoint of it for $t = \frac{1}{2}$ is called the geometric mean of A and B and denoted as $A\sharp B$. Audenaert recently proved that for commuting pairs of positive definite matrices A_i and $B_i, i = 1, \dots, m$ and for any unitary invariant norm $\|\cdot\|$

$$\left\| \sum_{i=1}^m A_i B_i \right\| \leq \left\| \left(\sum_{i=1}^m A_i^{1/2} B_i^{1/2} \right)^2 \right\| \leq \left\| \left(\sum_{i=1}^m A_i \right) \left(\sum_{i=1}^m B_i \right) \right\|.$$

We will talk about similar inequalities in the non-commuting case when matrix multiplication is replaced with the geometric mean. Also, Numerical counterexamples will be given for some related inequality questions. (Received September 26, 2017)