

1135-47-1235

**Joseph A Ball\*** (joball@math.vt.edu), Department of Mathematics, Virginia Tech, 225 Stanger Street, Blacksburg, VA 24061. *A Positivstellensatz for free noncommutative kernels.*

A **free nc kernel** is a function  $K$  of two matrix  $d$ -tuple arguments  $Z = (Z_1, \dots, Z_d)$  and  $W = (W_1, \dots, W_d)$  (with each  $Z_i$  say of size  $n \times n$  and each  $W_i$  of size  $m \times m$ ) coming from some domain  $\Omega$  with values in the space  $\mathcal{L}(\mathcal{A}^{n \times m}, \mathcal{B}^{n \times m})$  with  $\mathcal{A}$  and  $\mathcal{B}$  two given  $C^*$ -algebras (here  $d$  is fixed while  $n$  and  $m$  are arbitrary), subject to some additional compatibility conditions. Such a kernel  $K$  is said to be **completely positive (cp)** if the  $K(Z, Z)$  is a completely positive map for each  $Z \in \Omega$ . A **Positivstellensatz for free nc kernels** for a given pair of such kernels  $\mathfrak{Q}: \Omega \times \Omega \rightarrow \mathcal{L}(\mathcal{A}, \mathcal{L}(\mathcal{S}))_{\text{nc}}$  and  $\mathfrak{S}: \Omega \times \Omega \rightarrow \mathcal{L}(\mathcal{A}, \mathcal{L}(\mathcal{Y}))_{\text{nc}}$  is to obtain a **certificate** for when it is the case that  $\mathfrak{Q}(Z, Z)(I) \succ 0 \Rightarrow \mathfrak{S}(Z, Z)(I) \succeq 0$ . The talk will discuss the present status of this problem as well as its connections with free nc Nevanlinna-Pick interpolation and with other Positivstellensätze. (Received September 20, 2017)