1135-53-1046 **Curtis Pro*** (cpro@csustan.edu) and **Fred Wilhelm** (fred@math.ucr.edu). *Stability, Finiteness, and Dimension Four.*

In 1970, Cheeger showed that the class of n-dimensional Riemannian manifolds with fixed two-sided sectional curvature bounds, upper diameter bound, and positive lower volume bound contains, at most, finitely many diffeomorphism types. Subsequently, Grove, Petersen, Wu, and, independently, Perelman showed that, if $n \neq 4$, the same can be claimed without assuming a uniform upper bound on sectional curvature. Perelman's argument uses his Stability Theorem, which, in this setting, says: for any Gromov-Hausdorff converging sequence of n-manifolds all having a uniform lower bound on sectional curvature, upper bound on diameter, and positive lower bound on volume, almost all manifolds in this sequence must be homeomorphic. Combining this with Gromov's compactness theorem and, if $n \neq 4$, a topological result by Kirby and Siebenmann, diffeomorphism finiteness in this larger class follows. Perelman's argument begs the question: are almost all manifolds in this sequence also diffeomorphic? In this talk, I will outline the major points in joint work with Fred Wilhelm that show when $n = 4$, the answer is yes. In particular, Grove, Petersen, Wu/ Perelman's result also holds when $n = 4$, completing the generalization of Cheeger's theorem. (Received September 18, 2017)