

1145-03-1434

**Mojtaba Moniri\*** ([mojtaba.moniri@normandale.edu](mailto:mojtaba.moniri@normandale.edu)). *Addition with or without multiplication: algorithms, maximality, and near-linearity.*

We first mention two algorithms for a certain sequence of nonnegative integers, one which calculates in  $(\mathbb{Z}, +)$  in conjunction with the counting operator  $\#$  and the exponential substitution, and applies to any positive integer input. The other algorithm calculates in  $(\mathbb{Z}, +, \cdot)$ , and is more efficient when the input is a power of 2.

Next, let  $F$  be an ordered field,  $D$  a maximal discrete subring of  $F$ , and  $G$  a maximal discrete additive subgroup of  $F$ . We point out that although there are examples where  $F$  has elements of infinite distance to  $D$ , it can never realize any gaps of  $G$ . For countable  $F$ , the subgroup  $G$  can be constructed  $\Delta_2^0$  relative to  $F$ .

Finally we consider some nonstandard models  $M$  of weak arithmetic which have  $\mathbb{Z}$  as an additive direct summand. We present functions  $f, g : M \rightarrow M$  whose value at a sum minus sum of values is always 0 or 1 yet for some  $x, y, u, v \in M^{\geq 1}$ ,  $f(xy) < xf(y)$  and  $g(uv) > ug(v) + u - 1$ . (Received September 21, 2018)