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**Takayuki Kihara** and **Arno Pauly\*** (a.m.pauly@swansea.ac.uk). *A Turing ideal with positive outer measure.* Preliminary report.

We present a construction of a non-trivial Turing ideal  $\mathcal{I}$  such that for every oracle  $p$ , there exists some  $q \in \mathcal{I}$  which is random relative to  $p$ ; equivalently, such that  $\mathcal{I}$  has positive outer Lebesgue measure. That  $\mathcal{I}$  is non-trivial is witnessed by  $\emptyset' \notin \mathcal{I}$ . Our construction makes some mild assumptions about cardinal invariants of the continuum, which are independent of ZFC but follow from CH or MA.

A non-trivial Turing ideal with positive outer measure is the ultimate object that is not small in the sense of measure or dimension, that is closed under nice operations, and yet is not the whole. For example, we immediately obtain a proper subfield of the reals of positive outer measure, which is nonetheless algebraically closed, closed under the exponential function, every other computable operation, and even more so, closed under a set of operations with positive outer Wiener measure. (Received September 24, 2018)