

1145-03-586

**Martin D Davis\*** ([martin@eipye.com](mailto:martin@eipye.com)), 3360 Dwight Way, Berkeley, CA 94704. *Undecidable Propositions in Number Theory: Are All of Them Monsters?*

Foundational work in the 1930s yielded two different kinds of impossibility: algorithmic unsolvability (the non-existence of a uniform algorithm for determining the truth or falsity of each of an infinite class of assertions) and undecidable propositions (individual assertions that could neither be proved nor disproved in a particular given logical formalism). Whereas examples of the former kind are to be found in almost every branch of mathematics, results of the latter kind have so far had virtually no impact on mathematical practice. This is particularly striking in the case of number theory where Gödel and others have made much of the fact that such propositions can be found of a simple Diophantine form. However, when these are written explicitly, a monster results. Given that we know that new  $\Pi_1^0$  propositions become provable as ever stronger set-theoretic axioms are provided, it may well be the case that even such classic open problems as the Goldbach Conjecture and the Riemann Hypothesis may require set-theoretic methods. In his Gibbs address Gödel conjectured that this is indeed the case for RH. Fermat's Last Theorem has been proved, but it may not be provable in PA. Experts in weak arithmetics could seek models that falsify certain of these propositions. (Received September 10, 2018)