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**Frederic Meunier** and **Shira Zerbib\*** ([zerbib@umich.edu](mailto:zerbib@umich.edu)), University of Michigan,  
Department of Mathematics, Ann Arbor, MI 48109. *Envy-free division of a cake without the  
"hungry players" assumption.*

Consider  $n$  players having preferences over a rectangular cake, identified with the interval  $[0,1]$ . A classical theorem due to Stromquist ensures that under some conditions it is possible to divide the cake into  $n$  interval pieces and assign one piece to each player in an envy-free manner, such that no player strictly prefers a piece that has not been assigned to him. One of these conditions, which has been always considered as crucial, is that the players are "hungry": in every partition of the cake, every player prefers a non-empty piece. We prove that it is still possible to get an envy-free division even if this condition is not satisfied, when the number of players is prime or equal to 4. This was conjectured by Erel Segal-Halevi, who proved it for at most 3 players. The main step in our proof is a new combinatorial lemma in topology, which is reminiscent of the Sperner lemma: Instead of restricting the labels that can appear on each face of the simplex, the lemma considers labelings that enjoy a certain symmetry on the boundary. (Received September 24, 2018)