1145-11-1651 **Paul Pollack*** (pollack@uga.edu). *Popular values and popular subsets of Euler's φ-function.*

Let $N(m)$ denote the number of preimages of m under Euler's function. The number of integers that φ maps into $[1, N]$ can be shown to be $O(N)$, and so the function $N(m)$ is bounded on average. So it is maybe surprising that, as shown by Erdős in 1935, the individual values of $N(m)$ can be as large as m^c (for a constant $c > 0$) for infinitely many m. Erdős conjectured that c could be taken arbitrarily close to 1. In fact, under plausible conjectures on the distribution of smooth shifted primes, Pomerance showed in 1981 that $N(m) \geq m/L(m)^{1+o(1)}$ on an infinite sequence of m, where $L(x) = \exp(\log x \cdot \log_3 x / \log_2 x)$. Unconditionally, he proved that $N(m) \leq m/L(m)^{1+o(1)}$, whenever $m \to \infty$, so that $m/L(m)^{1+o(1)}$ describes the true "maximal order" of $N(m)$.

We discuss recent work counting preimages of subsets of $[1, N]$ (so that $N(m)$ tells the story for singleton sets). Two corollaries of this work are a conjecturally sharp upper bound for the second moment of $N(m)$, and a conjecturally sharp upper bound for the count of n with $\varphi(n)$ a square. (Received September 23, 2018)