We give metric theorems for the property of Borel normality for real numbers under the assumption of digit dependencies in their expansion in a given integer base. We quantify precisely how much digit dependence can be allowed such that, still, almost all real numbers are normal. Our theorem states that almost all real numbers are normal when at least slightly more than \( \log \log n \) consecutive digits with indices starting at position \( n \) are independent. As the main application, we consider the Toeplitz set \( T_P \), which is the set of all sequences \( a_1 a_2 \ldots \) of symbols from \( \{0, \ldots, b-1\} \) such that \( a_n \) is equal to \( a_{pn} \), for every \( p \) in \( P \) and \( n = 1, 2, \ldots \). Here \( b \) is an integer base and \( P \) is a finite set of prime numbers. We show that almost every real number whose base \( b \) expansion is in \( T_P \) is normal to base \( b \). In the case when \( P \) is the singleton set \( \{2\} \) we prove that more is true: almost every real number whose base \( b \) expansion is in \( T_P \) is normal to all integer bases.

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