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Yizhen Chen* (johnson.chen@prismsus.org). *The behavior of iterations of compositions of inversions preserving a circle.*

Let \mathcal{C} be a circle, and \mathcal{E} be a conic. Let $f_{\mathcal{E}} : \mathcal{C} \rightarrow \mathcal{C}$ be a homeomorphism such that line $xf(x)$ is always tangent to \mathcal{E} for $x \in \mathcal{C}$. Poncelet's porism states that if f^n has a fixed point, then f^n is the identity. We replace \mathcal{E} by a polygon, and study the behavior of a composition of inversions $f_A : \mathcal{C} \rightarrow \mathcal{C}$ where A is a point and line $xf_A(x)$ always passes through A for $x \in \mathcal{C}$. We found two different ways to convert a composition f of several inversions into a composition of two inversions. When f has no fixed points, we give a simple condition that f^n has a fixed point ($n > 1$), which is also equivalent to that f^n is the identity. When f has fixed points, one of the fixed points P has the property that $\lim_{n \rightarrow \infty} f^n(x) = P$ for every $x \in \mathcal{C}$ except the other fixed point. We found a simple criterion to determine which fixed point has this property for a composition f of m inversions f_{A_k} . For $f = f_C \circ f_B \circ f_A$ we have another simple criterion when all of A , B , and C are inside \mathcal{C} . Lastly, there is a conic $\mathcal{E}(A, B)$ such that $f_C \circ f_B \circ f_A$ has no fixed points if and only if C is inside it. We found several interesting properties of $\mathcal{E}(A, B)$. (Received August 19, 2018)