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Itai Seggev* (is+research@cs.hmc.edu). *Stability of ODEs and Limits Superior/Inferior as Reinforcing Concepts.*

Many physical systems are described by ordinary differential equations in which the short-time behavior (“transients”) are unimportant. The behavior of solutions at infinity describes the perceived response and stability of the system. The question naturally arises of how to compare different solutions that do not have a limit at infinity. Limits superior and inferior provide a useful quantitative framework for this question. The elementary notion of stability as a bounded response can easily be recast as a question about limits superior of the norm of solutions. For real-valued solutions $y(t)$, this condition can be formulated as

$$-\infty < \liminf_{t \rightarrow \infty} y(t) \leq \limsup_{t \rightarrow \infty} y(t) < \infty.$$

Different solutions can be compared by examining the appropriate limits superior and inferior. Such comparisons will be illustrated using common systems, such as a mass-spring system. This point of view can be introduced in a course on ordinary differential equations and modelling as a means of analyzing systems with periodic forcing functions. Alternatively, it can be introduced in a course on real analysis as a concrete application of limits superior and inferior. (Received September 08, 2018)