We study the Leibniz $n$-algebra $U_n(Lb)$, whose multiplication is defined via the bracket of a Leibniz algebra $L$ as $[x_1, \ldots, x_n] = [x_1, [\ldots, [x_{n-2}, [x_{n-1}, x_n]] \ldots]]$. We the simplicity of $U_n(L)$ when $L$ corresponds to a simple Lie algebra. An analogue of Levi’s theorem for Leibniz algebras in $U_n(Lb)$ is established and it is proven that the Leibniz $n$-kernel of $U_n(L)$ for any semisimple Leibniz algebra $L$ is the $n$-algebra $U_n(L)$. (Received September 24, 2018)