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Karen M. Lange* (karen.lange@wellesley.edu), Wellesley College, 106 Central St., Wellesley, MA 02481. *Different problems, common threads: Computing the difficulty of mathematical problems.*

Mathematics is filled with existence theorems such as “every vector space has a basis”. Such statements do not address how one goes about *finding* the known-to-exist object. The prior theorem naturally leads to the question “given a vector space, can we compute a basis for it?”. The answer to this “Basis Problem” is no, so we say that the problem is not “computable”.

We can further ask just how far from computable the Basis Problem is and what other problems have the same computational power. A natural way to compare the algorithmic difficulty of two problems is to determine whether having the ability to solve one allows for the solution of the other. Under this problem-reduction approach, two problems have the same computational power if each can be used to solve the other.

In this talk, we will explore the key ideas behind taking a “computable” perspective on mathematics and how this compares to an “existence” one. We will illustrate the problem-reduction approach using theorems from across mathematics. The overall structure of problem-difficulty is extremely rich and helps to illuminate what makes problems “tick”. Moreover, this approach has strong connections to calibrating exactly which axioms are needed to prove the original existence theorems. (Received September 16, 2019)