

1154-11-2358      **Lola Thompson\*** (lola.thompson@oberlin.edu) and **Harald Andres Helfgott**. *Summing  $\mu(n)$ : a faster elementary algorithm.*

Consider either of two related problems: determining the precise number  $\pi(x)$  of prime numbers  $p \leq x$ , and computing the Mertens function  $M(x) = \sum_{n \leq x} \mu(n)$ , where  $\mu$  is the Möbius function.

The two best algorithms known are the following:

1. An analytic algorithm (Lagarias-Odlyzko, 1987), with computations based on integrals of  $\zeta(s)$ ; its running time is  $O(x^{1/2+\epsilon})$ .
2. A more elementary algorithm (Meissel-Lehmer, 1959 and Lagarias-Miller-Odlyzko, 1985; refined by Deléglise-Rivat, 1996), with running time about  $O(x^{2/3})$ .

The analytic algorithm had to wait for almost 30 years to receive its first rigorous, unconditional implementation (Platt), which concerns only the computation of  $\pi(x)$ . Moreover, in the range explored to date ( $x \leq 10^{24}$ ), the elementary algorithm is faster in practice.

We present a new elementary algorithm with running time  $O(x^{3/5} \log x)$  for computing  $M(x) = \sum_{n \leq x} \mu(n)$ . The algorithm should be adaptable to computing  $\pi(x)$  and other related problems. This talk is based on joint work with Harald Andrés Helfgott. (Received September 17, 2019)