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John Greene* (jgreene@d.umn.edu), Department of Mathematics and Statistics, 1117 University Drive, Duluth, MN 55812, and **Mark Broderius** (brode076@d.umn.edu), Department of Mathematics and Statistics, 1117 University Drive, Duluth, MN 55812. *Primes in Lucas sequences*. Preliminary report.

A Lucas sequence is a sequence $U_n(P, Q)$ satisfying the recurrence

$$U_0 = 0, U_1 = 1, U_n = PU_{n-1} - QU_{n-2}, \text{ for } n \geq 2.$$

Special cases include the Mersenne numbers, $2^n - 1 = U_n(3, 2)$ and the Fibonacci numbers, $F_n = U_n(1, -1)$. A problem beyond current techniques is how often $U_n(P, Q)$ should be prime, for relatively prime integers P and Q . Heuristically, when n is prime, one might expect U_n to be prime with probability

$$\frac{1.8 \log(n)}{\log |U_n|},$$

and this estimate appears to be relatively accurate for $n \leq 1000$.

par However, some Lucas sequences appear to contain very few primes. For example, $U_n(5, 4)$ is prime only when $n = 2$, for rather obvious reasons. Here, we give families of P and Q for which we can prove that there are only finitely many primes in the corresponding Lucas sequence, and other families for which we conjecture that there are only finitely many primes. Some proofs seem surprisingly difficult. (Received September 03, 2019)