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It is a profound result of Hölder in 1887 that the Euler gamma-function cannot satisfy any nontrivial algebraic differential equation in \mathbf{C} . Hilbert, in his lecture addressed before the International Congress of Mathematicians at Paris in 1900 for his famous 23 problems, stated in Problem 18 that the Riemann zeta-function cannot satisfy any such a differential equation either.

In 2007, Markus showed that $\zeta(\sin(2\pi z))$ cannot satisfy such a differential equation with polynomial coefficients in Γ and its derivatives; he conjectured ζ itself cannot satisfy such a differential equation with polynomial coefficients in Γ and its derivatives either.

Thus, one wonders if there is a nontrivial polynomial $\mathcal{P}(u_0, u_1, \dots, u_m; v_0, v_1, \dots, v_n)$ such that

$$\mathcal{P}(\zeta, \zeta', \dots, \zeta^{(m)}; \Gamma, \Gamma', \dots, \Gamma^{(n)}) \equiv 0. \quad (1)$$

In this joint work with Dr. Jingbo Liu, we show that ζ and Γ cannot satisfy some differential equations generated through a family \mathfrak{F} of functions $\mathcal{F}(u_0, u_1, \dots, u_m; v_0, v_1, \dots, v_n)$ which are continuous in (u_0, u_1, \dots, u_m) with polynomial coefficients of (v_0, v_1, \dots, v_n) . (Received September 11, 2019)