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Stefan Steinerberger* (stefan.steinerberger@gmail.com), 10 Hillhouse Avenue, New Haven, CT 06511. *A Nonlocal Transport Equation Describing Roots of Polynomials Under Differentiation.*

Let p_n be a polynomial of degree n having all its roots on the real line distributed according to a smooth function $u(0, x)$. One could wonder how the distribution of roots behaves under iterated differentiation of the function, i.e. how the density of roots of $p_n^{(k)}$ evolves. We derive a nonlinear transport equation with nonlocal flux

$$u_t + \frac{1}{\pi} \left(\arctan \left(\frac{Hu}{u} \right) \right)_x = 0 \quad \text{on } \text{supp} \{u > 0\},$$

where H is the Hilbert transform. This equation has three very different compactly supported solutions: (1) the arcsine distribution $u(t, x) = (1 - x^2)^{-1/2} \chi_{(-1,1)}$, (2) the family of semicircle distributions

$$u(t, x) = \frac{2}{\pi} \sqrt{(T - t) - x^2}$$

and (3) a family of solutions contained in the Marchenko-Pastur law. (Received August 12, 2019)