

1154-35-785

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**Skrzeczkowski.** *Sensitivity equations for measure-valued solutions to transport equations.*

Transport type equations arise ubiquitously in the physical, biological and social sciences. They were used, for example, to model the dynamics of opinion formation and structured populations. Because of the natural setting of the space of measures for these equations, which allows for unifying discrete and continuous dynamics under the same framework, we consider the following transport equation in the space of bounded, nonnegative Radon measures  $\mathcal{M}^+(\mathbb{R}^d)$ :

$$\partial_t \mu_t + \partial_x(v(x)\mu_t) = 0.$$

We study the sensitivity of the solution  $\mu_t$  with respect to a perturbation in the vector field,  $v(x)$ . In particular, we replace the vector field  $v$  with a perturbation of the form  $v^h = v_0(x) + hv_1(x)$  and let  $\mu_t^h$  be the solution of

$$\partial_t \mu_t^h + \partial_x(v^h(x)\mu_t^h) = 0.$$

We derive a partial differential equation that is satisfied by the derivative of  $\mu_t^h$  with respect to  $h$ ,  $\partial_h(\mu_t^h)$ . We show that this equation has a unique very weak solution on the space  $Z$ , being the closure of  $\mathcal{M}(\mathbb{R}^d)$  endowed with the dual norm  $(C^{1,\alpha}(\mathbb{R}^d))^*$ . We also extend the result to the nonlinear case where the vector field depends on  $\mu_t$ , i.e.,  $v = v[\mu_t](x)$ . (Received September 10, 2019)