1154-37-1868 **David Sumner Lipham*** (`dlipham@aum.edu`). *Exploding endpoints and a positive entropy conjecture.*

For each $a \in (-\infty, -1)$ define $f_a : \mathbb{C} \to \mathbb{C}$ by $f_a(z) = \exp(z) + a$. Let $E(f_a)$ be the set of finite endpoints of maximal rays in the Julia set $J(f_a)$. Let $\dot{E}(f_a)$ and $\ddot{E}(f_a)$ be the sets of *escaping* and *fast escaping* points in $E(f_a)$.

In 1988, Mayer proved $E(f_a) \cup \{\infty\}$ is connected. In 1996, Kawamura Oversteegen and Tymchatyn proved $E(f_a)$ is homeomorphic to the *almost zero-dimensional* irrational Hilbert space $\mathfrak{E}_c$. In 2017, Alhabib and Rempe-Gillen proved $\dot{E}(f_a) \cup \{\infty\}$ and $\ddot{E}(f_a) \cup \{\infty\}$ are connected. Thus $\dot{E}(f_a)$ and $\ddot{E}(f_a)$ are recently discovered almost zero-dimensional spaces with one-point connectifications. In this talk we present joint work with Jan J. Dijkstra on new topological characteristics of these spaces. We conjecture $\ddot{E}(f_a) \simeq \mathbb{Q} \times \mathfrak{E}_c$ and $\dot{E}(f_a) \simeq \mathfrak{E}$, where $\mathfrak{E}$ is the rational Hilbert space.

Time permitting, we will also discuss a conjecture of Seidler and Kato linking the existence of positive entropy homeomorphisms to uncountable topological structure. We show an example by Tymchatyn re-opens the conjecture, and discuss other versions of the problem. (Received September 16, 2019)