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*Endpoint Sobolev continuity for the fractional maximal function.*

Let  $M_\beta$  denote the non-centered fractional maximal function on  $\mathbb{R}^d$ , where  $0 < \beta < d$ . A well known result of Kinnunen says that the  $L^p - L^q$  bounds satisfied by  $M_\beta$  are preserved at the derivative level, that is,

$$\|\nabla M_\beta f\|_q \leq \|\nabla f\|_p, \quad \text{for } \frac{1}{q} = \frac{1}{p} - \frac{\beta}{d}$$

and  $1 < p \leq \infty$ . Moreover, there are some instances in which the map  $f \mapsto |\nabla M_\beta f|$  satisfies strong bounds for the endpoint input space  $W^{1,1}$ . This is in contrast to the weak-type bounds satisfied by  $M_\beta$  for  $L^1$  functions.

As the map  $f \mapsto |\nabla M_\beta f|$  ceases to be sublinear, its boundedness from  $W^{1,p}$  to  $L^q$  does not immediately imply that is a continuous map between such function spaces. The first affirmative result in this direction was obtained by Luiro in the non-endpoint case  $1 < p \leq \infty$ .

For the endpoint space  $W^{1,1}$ , Madrid proved the continuity of such map in dimension  $d = 1$ . In this talk, we will present higher dimensional results. In particular, continuity from  $W^{1,1}(\mathbb{R}^d)$  to  $L^{d/(d-\beta)}(\mathbb{R}^d)$  holds if  $1 \leq \beta < d$  and if  $0 < \beta < 1$  for radial functions. (Received September 14, 2019)