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J. William Helton, Igor Klep, Scott McCullough and Jurij Volčič*

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The free semialgebraic set \mathcal{D}_f determined by a hermitian noncommutative polynomial $f \in M_d(\mathbb{C}\langle x, x^* \rangle)$ is the closure of the connected component of $\{X: f(X, X^*) \succ 0\}$ containing the origin. When L is a hermitian monic linear pencil, the free semialgebraic set \mathcal{D}_L is called a free spectrahedron. Since it is the feasible set of a linear matrix inequality (LMI), it is evidently convex. Conversely, it is well-known that every convex free semialgebraic set is a free spectrahedron. This talk presents a solution to the basic problem of determining those f for which \mathcal{D}_f is convex. A consequence is an effective probabilistic algorithm that not only determines if \mathcal{D}_f is convex, but if so, produces its LMI representation. Of independent interest is a subalgorithm based on a Nichtsingulärstellensatz: given a linear pencil L and a free spectrahedron \mathcal{D} , it determines if L takes only invertible values on the interior of \mathcal{D} . Lastly, if \mathcal{D}_f is convex and $f \in \mathbb{C}\langle x, x^* \rangle$ is irreducible, then f is quadratic. (Received August 24, 2019)