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Classifying solutions for an important functional equation in Complex Analysis.

Let \( \varphi : \mathbb{D} \to \mathbb{D} \) be a non-constant rational map in the Hardy space, and let \( C_\varphi \) act on the Hardy space. Set

\[
\sigma(z) = \frac{1}{\varphi^{-1}(\sqrt{z})}, \quad \psi(z) = \frac{z \sigma'(z)}{\sigma(z)}, \quad \text{and} \quad \varphi(\infty) = \lim_{|z| \to \infty} \varphi(z).
\]

Then,

\[
(C_\varphi^* f)(z) = \frac{f(0)}{1 - \varphi(\infty) z} + \sum \psi(z)f(\sigma(z))
\]

where the sum is taken over the branches of \( \sigma \). The kernel of \( C_\varphi^* \) is a subset of \( H^2 \) such that if \( f \in \ker(C_\varphi^*) \), then \( C_\varphi^*(f) = 0 \). It is very important but also turns out to be very hard to classify the kernel of \( C_\varphi^* \). According to the previous research, the following functional equation point out a possible way to classify the kernel of \( C_\varphi^* \).

\[
U(z)f(z) + zU'(z)f(U(z)) = 0
\]

where \( U \) is always represented by one of following 5 forms: \(-z, \frac{m}{z}, m - z, \frac{-z}{1 + m z}, \frac{-m + z}{1 + t z}\), where \( m, t \in \mathbb{R} \) and \( m \neq 0, t \neq 0 \).

To classify the kernel of \( C_\varphi^* \), the research was conducted to investigating the solutions of each case of \( u \) for the functional equation. (Received September 17, 2019)