Preservation properties are a tool for separating the reverse mathematical strength of various statements. As an example, if $I$ is a Turing ideal and $X$ is a set outside $I$, then there is an ideal $J$ containing $I$ but omitting $X$ and which models WKL$_0$. The same holds with RT$_2^2$ in place of WKL$_0$, but this fails for RT$_3^2$, thus showing that WKL$_0$ and RT$_2^2$ do not prove RT$_3^2$.

In fact, for both WKL$_0$ and RT$_2^2$, the above holds not just for a single set $X$, but for countably many sets simultaneously. In both cases, the proofs for one set and for countably many sets are more or less the same. It turns out there’s a reason for this: any reverse mathematical principal (of the appropriate form) which can be satisfied while avoiding a single set can be satisfied while avoiding countably many.

This is an example of a relationship between preservation properties. We investigate similar relationships between various preservation properties. (Received September 14, 2020)