A cohesive power of a computable structure is an effective analog of an ultrapower where a cohesive set acts as an ultrafilter. We study cohesive powers of computable copies of $\omega$, which are computable linear orders that are isomorphic to $(\mathbb{N}, <)$, but not necessarily by computable isomorphisms.

Every cohesive power of the standard presentation of $\omega$ has order-type $\omega + \zeta \eta$, which is expected because $\omega + \zeta \eta$ is the familiar order-type of countable non-standard models of PA. We show that it is possible for cohesive powers of computable copies of $\omega$ to exhibit a variety of order-types:

- There is a computable copy of $\omega$ with a cohesive power of order-type $\omega + \eta$.

- For every finite, non-empty $X \subseteq \mathbb{N} \setminus \{0\}$ (thought of as a set of finite order-types), there is a computable copy of $\omega$ with a cohesive power of order-type $\omega + \sigma(X)$. Here $\sigma$ denotes the shuffle operation.

- For every $X \subseteq \mathbb{N} \setminus \{0\}$ that is either $\Sigma^0_2$ or $\Pi^0_2$, there is a computable copy of $\omega$ with a cohesive power of order-type $\omega + \sigma(X \cup \{\omega + \zeta \eta + \omega^*\})$.

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