1163-05-1553 **Noah Kravitz*** (nkravitz@princeton.edu). *Barely lonely runners and very lonely runners: a refined approach to the Lonely Runner Problem.*

We introduce a sharpened version of the well-known Lonely Runner Conjecture of Wills and Cusick. Let $\|x\|$ denote the distance from a real number x to the nearest integer. For each set of positive integer speeds $v_1, \dots, v_n$, define the associated maximum loneliness

$$\mathrm{ML}(v_1, \dots, v_n) = \max_{t \in \mathbb{R}} \min_{1 \le i \le n} \|tv_i\|.$$

The Lonely Runner Conjecture asserts that

$$\mathrm{ML}(v_1, \dots, v_n) \ge \frac{1}{n+1}$$

for all choices of $v_1, \dots, v_n$. If this conjecture is true, then the quantity $1/(n+1)$ is the best possible, for there are known equality cases with $\mathrm{ML}(v_1, \dots, v_n) = 1/(n+1)$. A natural but hitherto unasked question is: If $v_1, \dots, v_n$ satisfy the Lonely Runner Conjecture but are not an equality case, must $\mathrm{ML}(v_1, \dots, v_n)$ be uniformly bounded away from $1/(n+1)$?

We conjecture that, surprisingly, the answer is yes. More precisely, we conjecture that for each choice of $v_1, \dots, v_n$, we have either $\mathrm{ML}(v_1, \dots, v_n) = s/(ns+1)$ for some $s \in \mathbb{N}$ or $\mathrm{ML}(v_1, \dots, v_n) \ge 1/n$. Our main results are: confirming this stronger conjecture for $n \le 3$; and confirming it for $n = 4$ and $n = 6$ in the case where one speed is much faster than the rest. (Received September 15, 2020)