Splines are a fundamental tool across applied math and analysis, used in fields from computer graphics to data interpolation. We work with an algebraic-combinatorial generalization of splines, in which we start with an edge-labeled graph. A spline on this graph is a labeling of vertices so that adjacent vertex-labels differ by a multiple of the edge-label. We will study particular families of graphs (especially those dual to planar triangulations) in order to find a basis (and/or the dimension) of the space of splines when using polynomial labels of degree at most 2. This is part of a longstanding open problem sometimes called the “upper-bound conjecture”—which, curiously, is completely solved in the case of degree 4 or larger, has a well-known and well-tested conjectural formula in degree 3, and completely mysterious in degree 2. (Received September 15, 2020)