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Blake Dunshee and **Mark Ellingham*** (mark.ellingham@vanderbilt.edu). *A Fano framework for graph embeddings.*

One useful representation of embedded graphs is by *graph-encoded maps* or *gems*, which represent a cellularly embedded graph as a cubic graph with a proper 3-edge-colouring. A number of graph embedding properties, such as orientability or bipartiteness, correspond to parity conditions on closed walks in the gem. Using this correspondence we can map seven natural embedding properties to points in the Fano plane (the projective plane of order 2), in such a way that there is a theorem associated with every set of three properties. The type of theorem depends on whether the associated points lie on a line in the Fano plane or not. The seven properties also correspond to the existence of certain orientations or signings of the embedded medial graph. This allows us to construct theorems on whether embeddings have a property after taking partial duals or partial Petrie duals (twists) with respect to certain edges. This generalizes work by Huggett and Moffatt and by Deng and Jin. We also discuss other interpretations of the seven properties, and extensions to other properties. (Received September 10, 2020)