Any graph $G$ with chromatic number $k$ can be constructed by iteratively performing certain graph operations on a sequence of graphs starting with $K_k$, resulting in a variety of Hajos-type constructions for $G$. Finding such a construction for a given graph or family of graphs is a challenging task. In this talk, I will present our results which imply that for a graph $G$ with a highly-connected neighborhood complex, the end behavior of the construction is quite restricted. Time permitting I will share results of computational experiments. (Received September 13, 2020)