Congruences of Fourier coefficients of modular forms have long been an object of central study. By comparison, the arithmetic of other expansions of modular forms, in particular Taylor expansions around points in the upper-half plane, has been much less studied. Recently, Romik made a conjecture about the periodicity of coefficients around the point $i$ for the classical Jacobi theta function. Here we prove this conjecture and generalize the phenomenon observed by Romik to a general class of modular forms of half-integral weight. (Received September 13, 2020)