An old theorem of Berggren says that there exist three square matrices $M_{1}, M_{2}, M_{3}$ of size 3 with integer coefficients which satisfy the following property; every Pythagorean triple, that is, an integral zero of the Pythagorean form $x^{2}+y^{2}-z^{2}$, is obtained in a unique way by multiplying either $(3,4,5)$ or $(4,3,5)$ by $M_{d_{1}} \cdots M_{d_{k}}$ for some $d_{j} \in\{1,2,3\}$. More recently, a similar theorem was found for the integral zeros of several other indefinite quadratic forms. In this talk we will apply these theorems to obtain Lagrange spectra arising from intrinsic Diophantine approximation of circles. A large part of this talk is based on joint work with Dong Han Kim. We will also highlight contributions of undergraduate collaborators. (Received September 14, 2020)

