An atomic integral domain, $R$, is said to be a half-factorial domain (HFD) if given the irreducible factorizations

$$\alpha_1\alpha_2\cdots\alpha_n = \beta_1\beta_2\cdots\beta_m$$

then $n = m$.

If $R$ is an HFD with quotient field $K$ then the boundary map is a function $\partial_R : K^* \to \mathbb{Z}$ such that $\partial_R(xy) = \partial_R(x) + \partial_R(y)$ for all $x, y \in K^*$ and $\partial_R(\pi) = 1$ for all $\pi \in \text{Irr}(R)$ ($\partial_R$ is well-defined precisely when $R$ is an HFD).

The boundary map has been used with some success to determine stability of the half-factorial property in some special ring extensions, and in this talk we will describe how the boundary map can play a utilitarian role in investigating (complete) integral closures of HFDs. We will also explore the open question as to whether nonunits are always contained in the support of the boundary function in the case of atomic integral extensions. (Received August 16, 2020)