The Langlands program, proposed by Robert Langlands in 1960s, unifies many questions in number theory and representation theory, and has found significant applications to solving classical Diophantine equations. Its geometric version, formulated by Drinfeld and Laumon in 1980s, enlarges the scope of the Langlands philosophy and makes it contact with other subjects such as physics. Interestingly, in recent years, some ideas from the geometric theory also inspire and lead developments of the traditional arithmetic theory and related problems. (Received September 16, 2020)