In a series of papers joint with Sierra and Stafford, we have studied the birational geometry of projective noncommutative surfaces containing a smooth elliptic curve as a divisor. In particular, we have defined notions of blowing up a point and blowing down a curve of self-intersection -1 in this setting. In this talk we explain how these notions can be used to give two explicit birational transformations modeled on important examples in the commutative case. The first shows that a quadric surface blown up at a point can then be blown down twice to obtain a noncommutative projective plane. The second is a version of the Cremona Transform, which blows up three points on the projective plane and then blows down the three new lines of self-intersection -1 that appear. Both constructions are based on a result of independent interest, which shows how to characterize surfaces that are a blowup of the projective plane at two points, in terms of intersection theory. (Received September 03, 2020)