CR manifolds are smooth manifolds which generalize the notion of boundaries of domains in $\mathbb{C}^n$. Many abstract CR manifolds cannot be globally embedded as CR submanifolds of $\mathbb{C}^n$ for any $n$ (Burns and Epstein 1990) but there are few well-known examples of non embeddable CR manifolds. The Rossi sphere, which is defined as the regular sphere $S^2 \subset \mathbb{C}^2$ endowed with a perturbed CR structure, is the canonical example of a non-embeddable abstract CR manifold. A modern result states that one can detect CR-embeddability for certain CR manifolds by analyzing the bottom of the spectrum of the Kohn Laplacian. We study similar questions for spherical 3-manifolds that are obtained by taking the quotient of the sphere by left actions of finite subgroups of $SU(2)$. Using spectral-theoretic techniques, we prove that the quotient of the Rossi sphere by the antipodal map is CR-embeddable. We further generalize this result, proving that a quotient of the Rossi sphere which can be understood as a lens space $L(p, p - 1)$ with a perturbed CR structure is CR-embeddable if and only if $p$ is even. This characterization gives an infinite family of explicit examples of non-embeddable CR manifolds. (Received September 15, 2020)