Existence of solutions to a class of boundary value problems for Caputo fractional q-difference inclusions of the form

\[
\begin{cases}
(CD_q^\alpha u)(t) \in F(t, u(t)), & t \in I := [0, T], \\
L(u(0), u(T)) = 0,
\end{cases}
\]

are proved. Here, \( q \in (0, 1) \), \( \alpha \in (0, 1] \), \( T > 0 \), \( F : I \times \mathbb{R} \to \mathcal{P}(\mathbb{R}) \) is a multivalued map, \( \mathcal{P}(\mathbb{R}) \) is the family of all nonempty subsets of \( \mathbb{R} \), \( CD_q^\alpha \) is the Caputo fractional q-difference derivative of order \( \alpha \), and \( L : \mathbb{R}^2 \to \mathbb{R} \) is a continuous function.

The technique of proof involves using set-valued analysis, fixed point theory, and the method of upper and lower solutions. This appears to be the first time the method of upper and lower solutions has been applied to Caputo q-fractional difference equations. (Received September 10, 2020)