The scaling properties of many fluid models imply scaling invariant initial data do not exist within the classical Lebesgue space framework. Weighted spaces provide a large and powerful setting to analyze large scaling invariant and generic solutions. We survey results in this direction on global existence and eventual regularity of infinite energy solutions for the Navier-Stokes equations with non-decaying data (joint work with Igor Kukavica and Tai-Peng Tsai) as well as global existence of generic and special solutions to the critical dissipative surface quasi-geostrophic equations (joint work with Dallas Albritton). (Received September 14, 2020)