Modeling the effects of trait-mediated dispersal on coexistence of mutualists.

We analyse positive solutions \((u, v)\) to the steady state reaction diffusion system:

\[
\begin{align*}
-\Delta u &= \lambda u(1 - u); \quad \Omega \\
-\Delta v &= \lambda rv(1 - v); \quad \Omega \\
\frac{\partial u}{\partial \eta} + \sqrt{\lambda g(v)}u &= 0; \quad \partial \Omega \\
\frac{\partial v}{\partial \eta} + \sqrt{\lambda h(u)}v &= 0; \quad \partial \Omega
\end{align*}
\]

where \(\lambda > 0, r > 0\) are parameters and \(g, h \in C^1([0, \infty), (0, \infty))\) are decreasing functions. This system models the steady states of two species living in a habitat where the interaction is limited to the boundary. Here, \(\lambda\) is directly proportional to the size of the habitat and we will study the ranges of \(\lambda\) where coexistence and nonexistence occurs. Namely, we will consider three cases: (a) \(E_1(1, g(0)) = E_1(r, h(0))\), (b) \(E_1(1, g(0)) > E_1(r, h(0))\), (c) \(E_1(1, g(0)) < E_1(r, h(0))\). Here \(E_1(r, K)\) denotes the principal eigenvalue of:

\[
-\Delta z = rEz; \quad \Omega; \quad \frac{\partial z}{\partial \eta} + K\sqrt{E}z = 0; \quad \partial \Omega.
\]

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