Integrable hierarchies arise by starting with a differential equation that models a real-life process and constructing a system of infinitely many differential equations that can be solved simultaneously. Some classical examples of integrable hierarchies are the KP, KdV, and Toda hierarchies. Solutions to such hierarchies can be studied algebraically using tools such as Darboux transformations.

I will begin this talk by introducing the Toda hierarchies, with emphasis on the extended bigraded Toda hierarchy (EBTH). Darboux transformations for the EBTH were determined by Li and Song in 2016. I will show that the action of these Darboux transformations on solutions to the EBTH is given by a vertex operator. As a consequence of this result, I will derive a formula for generalized Fay identities of the EBTH.

This is joint work with B. Bakalov. (Received September 07, 2020)