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Dimension of the exceptional set in the Aronszajn–Donoghue theorem for finite rank perturbations.

The classical Aronszajn–Donoghue theorem states that for a rank one perturbation of a self-adjoint operator (by a cyclic vector) the singular parts of the spectral measures of the original and perturbed operators are mutually singular. As simple direct sum type examples show, this result does not hold for finite rank perturbations. However, the set of exceptional perturbations is pretty small.

Namely, for a family of rank $d$ perturbations $A_\alpha := A + B\alpha B^*, B : \mathbb{C}^d \to \mathcal{H}$, with Ran $B$ being cyclic for $A$, parametrized by $d \times d$ Hermitian matrices $\alpha$, the singular parts of the spectral measures of $A$ and $A_\alpha$ are vector mutually singular for all $\alpha$ except for a small exceptional set $E$.

It was shown earlier, that $E$ is a subset of measure zero of the space of $d \times d$ Hermitian matrices. We now learn that the set $E$ has small Hausdorff dimension, $\dim (E) \leq d^2 - 1$.

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