Let $B = (b_1, \ldots, b_N)$ be a row vector of analytic functions such that $\|B(z)\|^2 = \sum_j |b_j(z)|^2 < 1$ for all $z$ in the open unit disc $\mathbb{D}$. The de Branges-Rovnyak space $H(B)$ is defined by the reproducing kernel $\frac{1-B(z)B(w)^*}{1-zw}$. Assume that $M_z$ acts boundedly on $H(B)$ and let $\mathcal{M}$ be the largest subspace of $H(B)$ such that $M_z$ is isometric on $\mathcal{M}$. We show that all $b_i$'s are rational, if and only if $\mathcal{M}^\perp$ is finite dimensional, and that the degree of the rational tuple $B$ equals the dimension of $\mathcal{M}^\perp$. Furthermore, in this case $\mathcal{M}^\perp$ is invariant for the backward shift $L$, and there is a constant $c$ such that $a = c^2$ satisfies $|a|^2 + \|B\|^2 = 1$ a.e. on $\partial \mathbb{D}$. Here $p$ and $q$ are the characteristic polynomials of $M_z^*|\mathcal{M}^\perp$ and $L|\mathcal{M}^\perp$, and $\tilde{r}(z) = z^n r(1/z)$, $n = \dim \mathcal{M}^\perp$.

If $B$ is rational and if $M_z : H(B) \to H(B)$ is a $2m$-isometric operator, then all the zeros of $p$ lie in the unit circle and the norm of the functions in $H(B)$ can be expressed by use of $m$-th order local Dirichlet integrals. (Received September 10, 2020)