Dustin Arendt, Matthew Broussard* (matthew.broussard@wsu.edu), Bala Krishnamoorthy and Nathaniel Saul. Steinhaus Filtration and Stable Paths in the Mapper.

Two key concepts of topological data analysis are persistence and the Mapper. Persistence uses a sequence of objects built on data called filtration. Mapper produces insightful summaries of data, and is applied widely.

We define a new filtration called the cover filtration built from a single cover using a generalized Steinhaus distance (which generalizes Jaccard distance). We prove a stability result: cover filtrations of two covers are $a/m$ interleaved when bottleneck distance between covers $\leq a$ and size of smallest set in either cover is $m$. We show our construction is equivalent to Čech filtration under some settings. Also, Vietoris-Rips filtration completely determines the cover filtration in all cases. We develop a theory for stable paths within this filtration.

We show our framework can be employed in many applications when a metric is not given but a cover is readily available. We present a new model for recommendation systems using cover filtration. Stable paths on a movies data set give gentle transitions from one genre to another. In explainable machine learning, we apply Mapper for model induction, and provide explanations of a predictive model using paths between subsets of images.

A preprint is available at https://arxiv.org/abs/1906.08256. (Received September 12, 2020)