Chapter 2

CBMS2005 Special Projects

Each CBMS survey accepts proposals for special projects from various professional society committees. Special projects chosen for one CBMS survey might, or might not, be continued in the next CBMS survey. This chapter presents data from the special projects of CBMS2005:

- The mathematical education of pre-college teachers (Tables SP.1 to SP.10)
- Academic resources available to undergraduates (Tables SP.11 to SP.15)
- Dual enrollments in mathematics (Tables SP.16 and SP.17)
- Mathematics and general education requirements (Table SP.18)
- Requirements in the national major in mathematics and statistics (Tables SP.19 to SP.24)
- Assessment in mathematics and statistics departments (Table SP.25).

<u>Terminology</u>: Recall that in CBMS2005, the term “mathematics department” includes departments of mathematics, applied mathematics, mathematical sciences, and departments of mathematics and statistics. Experience shows that mathematics departments may offer a broad spectrum of courses in mathematics education, actuarial science, and operations research as well as in mathematics, applied mathematics, and statistics. Computer science courses are sometimes also offered by mathematics departments. The term “statistics department” refers to departments of statistics or biostatistics that offer undergraduate statistics courses. Courses and majors from separate departments of computer science, actuarial science, operations research, etc., are not included in CBMS2005. Departments are classified by highest degree offered. For example, the term “masters-level department” refers to a department that offers a masters degree but not a doctoral degree.

Tables SP.1 to SP.10: The Mathematical Education of Pre-college Teachers

In 2001, the American Mathematical Society (AMS) and the Mathematical Association of America (MAA) jointly published a CBMS study entitled *The Mathematical Education of Teachers* [MET] that made recommendations concerning the amount and kind of undergraduate mathematics and statistics that pre-service teachers should study. MET also called for closer collaboration between mathematicians and mathematics educators in the design of the undergraduate mathematics and statistics courses that pre-service teachers take. CBMS2000 provided baseline data about the extent to which the MET recommendations were already in place in fall 2000 and CBMS2005 provided five-year-later data to track further implementation of the MET report.

Table SP.1 shows that, in fall 2005, about 87% of mathematics departments and 44% of statistics departments reported belonging to a college or university that offered a teacher certification program for some or all of grades K–8. This compares to percentages of 84% for mathematics departments and 58% for statistics departments in fall 2000. The meaning of the fourteen point drop among statistics departments is not clear.

TABLE SP.1 Percentage of mathematics departments and statistics departments whose institutions offer a certification program for some or all of grades K–8, by type of department, in fall 2005. (Data from fall 2000 in parentheses).

	Percentage whose institutions have a K-8 teacher certification program
Mathematics Departments	
Univ (PhD)	78 (72)
Univ (MA)	92 (87)
Coll (BA)	88 (85)
Total Math Depts	**87 (84)**
Statistics Departments	
Univ (PhD)	40 (58)
Univ (MA)	59 (63)
Total Stat Depts	**44 (58)**

At the time of CBMS2000, teacher certification programs were almost entirely limited to four-year colleges and universities. By fall 2005 that had changed. Table SP.2 shows the percentages of public two-year colleges with programs allowing three types of students to complete their entire mathematics certification requirements at the two-year college. The three types of students mentioned in the table are undergraduates without a bachelors degree (called “pre-service teachers”), in-service teachers who already have certification in some other subject, and people who leave a first career to enter a second career in pre-college teaching (called “career-switchers”). The percentages in Table SP.2 are not large, but given the large number of two-year colleges in the U.S., it is clear that two-year colleges could make a major contribution to educating the next generation of teachers. Table SP.2 shows that two-year college credentialing programs tended to focus on producing K–8 teachers.

TABLE SP.2 Percentage of mathematics programs at public two-year colleges (TYCs) having organized programs that allow various types of pre- and in-service teachers to complete their entire mathematics course or licensure requirements, in fall 2005.

	Percentage of TYCs with an organized program in which students can complete their entire mathematics course or licensure requirements
Pre-service elementary teachers	30
Pre-service middle-school teachers	19
Pre-service secondary teachers	3
In-service elementary teachers	16
In-service middle school teachers	15
In-service secondary teachers	2
Career-switchers aiming for elementary teaching	19
Career-switchers aiming for middle school teaching	14
Career-switchers aiming for secondary teaching	6

To what extent did mathematics and statistics departments in four-year colleges and universities cooperate with their schools of education in teacher certification programs in fall 2005? One mark of such cooperation is for the department to have a seat on the committee that governs the certification program. Table SP.3 shows that about 80% of all mathematics departments were represented on that governing committee in fall 2005 (with considerable variation by type of department). Fewer statistics departments (about 28%) had members on the governing committees. Table SP.3 shows that the fall 2005 percentages were substantially larger than the corresponding percentages in CBMS2000, which reported 69% for mathematics departments and 0% for statistics departments (see CBMS2000 Table PSE.2).

Another mark of a department's involvement in K-8 teacher education is the existence of special mathematics (or statistics) courses or course sequences designed for K-8 pre-service teachers. Table SP.3 shows that the percentage of mathematics departments having such sequences rose from 77% in fall 2000 to 86% in fall 2005. The percentage of statistics departments with a special course for pre-service K-8 teachers was smaller in fall 2005 than the percentage for mathematics departments, but was higher than in fall 2000.

TABLE SP.3 Percentage of mathematics and statistics departments in universities and four-year colleges offering K-8 certification programs that are involved in K–8 teacher certification in various ways, by type of department, in fall 2005. (Data from fall 2000 in parentheses).

	Percentage of departments in schools offering K–8 certification programs that		
	Have a department member on the certification program's control committee	Offer a special course or course sequence for K-8 teachers	Designate special sections of regular courses for K-8 teachers
Mathematics Departments			
Univ (PhD)	58 (63)	81 (79)	31 (11)
Univ (MA)	86 (74)	96 (92)	45 (13)
Coll (BA)	82 (68)	85 (73)	21 (4)
Total Math Depts	80 (69)	86 (77)	25 (7)
Statistics Departments			
Univ (PhD)	29 (0)	11 (4)	0 (0)
Univ (MA)	25 (0)	33 (0)	0 (0)
Total Stat Depts	28 (0)	16 (4)	0 (0)

Note: 0 means less than one-half of 1%.

Table SP.4 shows a clear trend away from special mathematics courses for pre-service teachers in two-year college curricula, with the percentage of two-year colleges offering such courses in fall 2005 being less than one-fourth of the corresponding percentage reported for fall 2000 by CBMS2000. This decrease stands in marked contrast to the situation in four-year colleges and universities.

TABLE SP.4 Percentage of public two-year colleges (TYCs) that are involved with K-8 teacher preparation in various ways, in fall 2005.

	Percentage of TYCs
Assign a mathematics faculty member to coordinate K–8 teacher education in mathematics	38
Offered a special mathematics course for preservice K–8 teachers in 2004–2005 or 2005–2006	11
Offer mathematics pedagogy courses in the mathematics department	9
Offer mathematics pedagogy courses outside of the mathematics department	10

How many mathematics courses were required for a student seeking K–8 certification in fall 2005? That is a complicated question because of the wide variety of certification programs in the U.S. In fall 2005, some colleges and universities offered a single-track program for K–8 certification, while others divided K–8 certification into two sub-tracks (one for early grades and one for later grades), and still others further subdivided their later-grades track into discipline-specific later-grade certification programs. (In a discipline-specific later-grades program, a student might become certified to teach in some cluster of disciplines, say mathematics and science, in the later grades.) CBMS2005 addressed that diversity by dividing universities with K–8 certification programs into those that had a single set of mathematics requirements for K–8 certification, and those that had different mathematics requirements for early and later grade certification.

But even the meaning of "early grades" and "later grades" is complicated, because in fall 2005, different states, colleges, and universities divided K–8 certification in different ways. Some, for example, had an undivided K–8 certification, others put grades 4, 5, 6, 7, and 8 together in a single certification category, and still others put only grades 6, 7, and 8 together. In an attempt to make a single questionnaire fit all of the certification patterns, the CBMS2005 questionnaire defined the term "early grades certification" to mean the certification that included grades K–3, and defined the term "later grades certification" to be the certification that included grades 5 and 6.

Table SP.5 shows that the majority (56%) of departments with K–8 certification programs do not distinguish between early and later grades in terms of mathematics requirements, and also shows how many mathematics courses are required for various certifications. Comparisons with CBMS2000 data are possible, at least for programs that have different requirements for early and later grades. In each type of mathematics department, the number of mathematics courses required for K–8 teacher certification rose between fall 2000 and fall 2005. Chapter 2 of *The Mathematical Education of Teachers* recommended that K–3 teachers take at least nine semester hours of mathematics, which translates into three one-semester courses, and that prospective teachers of the middle grades should take at least 21 semester hours, which translates into seven semester courses. For CBMS2005, all reported data on course requirements were translated into semester courses, and Table SP.5 shows that while MET's course recommendations had not been completely implemented by fall 2005, the nation was closer to them than in the base-year study in fall 2000.

TABLE SP.5 Among all four-year colleges and universities with K-8 certification programs, the percentage that have different requirements for early grades (K–3) certification and for later grades (including 5 and 6) certification in terms of semester courses, including the number of semester courses required, and the percentage that have the same requirements for their combined K-8 certification program, including the number of courses required, in fall 2005. Also the average number of semester mathematics department courses required for various teacher certifications in those colleges and universities offering K–8 certification programs, by certification level and type of department, in fall 2005. (Data for fall 2000 in parentheses).

	Having different mathematics requirements for early & later grades certification		Having the same mathematics requirements for early & later grades certification
Percentage of mathematics departments with K-8 certification programs	44%		56%
	Percentage of departments with K-8 certification programs that require various numbers of mathematics courses		Percentage of departments with K-8 certification programs that require various numbers of mathematics courses
Number of mathematics courses required for certification	for early grades	for later grades	for all K-8 grades
0 required	11 (8)	16 (7)	4 (na)
1 required	17 (17)	7 (12)	26 (na)
2 required	31 (45)	5 (42)	37 (na)
3 required	17 (14)	2 (12)	22 (na)
4 required	17 (11)	11 (10)	11 (na)
5 or more required	8 (6)	58 (18)	0 (na)
Type of mathematics department	Avg number of courses required	Avg number of courses required	Avg number of courses required in combined K-8 certification program
Univ(PhD)	3.3 (2.2)	5.5 (2.5)	2.4 (na)
Univ(MA)	3.3 (3.3)	6.9 (4.1)	2.5 (na)
Coll(BA)	2.5 (2.3)	5.3 (2.8)	2 (na)
All mathematics departments	2.7 (2.4)	5.6 (3)	2.1 (na)

In fall 2005, which mathematics courses did pre-service K–8 teachers take? Table SP.6 records departmental responses to the question "In your judgment, which three of the following courses in your department are most likely to be taken by pre-service K–8 teachers?" The responses recorded in SP.6 can be compared with Table PSE.5 of CBMS2000. It would have been desirable to pose a more precise question, such as "Of all students receiving certification for part or all of grades K–8 between July 1, 2004 and June 30, 2005, what percentage actually took each of the following courses?" The CBMS2005 project directors decided that the data retrieval work required for a department to answer the more precise question would cut into CBMS2005 survey response rates in a major way, so the less precise question was used. This may limit the utility of Table SP.6. With that caveat in place, Table SP.6 suggests some conclusions. It suggests that in fall 2005 there were clear differences between the mathematical expectations for early and later-grade certification programs, that the mathematics requirements for K–3 certification seemed to center on a multi-term course (e.g., a two-semester sequence) for elementary education majors and a course in College Algebra, and that the mathematics requirements for later-grades certification seemed to focus on Calculus, Geometry, and Elementary Statistics. (See Table SP.8, below, for a discussion of when pre-service K–8 teachers begin their mathematics and statistics studies.)

TABLE SP.6 Among mathematics departments at four-year colleges and universities having different requirements for early and later grades certification, the percentage identifying a given course as one of the three mathematics courses most likely to be taken by pre-service teachers preparing for K–3 teaching or for later grades teaching (including 5 and 6) by type of department, in fall 2005.

	Most likely for K–3 certification			Most likely for later grades certification		
Among Mathematics Departments With Different Early and Later Grades Requirements	Univ (PhD) Math	Univ (MA) Math	Coll (BA) Math	Univ (PhD) Math	Univ (MA) Math	Coll (BA) Math
Multi-term course for elementary education majors	59	70	64	28	47	38
Single term course for elementary education majors	21	37	33	16	10	12
College algebra	41	40	56	21	40	23
Precalculus	15	6	46	13	13	15
Intro to mathematical modeling	5	0	0	8	0	0
Mathematics for liberal arts	28	30	25	8	7	2
Finite mathematics	23	7	15	10	7	8
Mathematics history	5	0	0	31	23	18
Calculus	21	6	12	64	50	77
Geometry	10	24	0	43	47	53
Elementary Statistics	31	26	27	41	44	55

Yet another mark of departmental involvement in K–8 teacher education is the appointment of a department member to coordinate the program. Table SP.4 shows that about 38% of two-year colleges appointed such a coordinator in fall 2005, up from 22% in fall 2000 reported in CBMS2000 Table PSE.3. CBMS2005 posed a different question to four-year mathematics departments in fall 2005. Four-year mathematics departments that offered multiple sections of their elementary mathematics education course were asked whether they appointed a department member to coordinate the multi-section course. Table SP.7 shows that the percentage varied from 90% among doctoral departments that offered multiple sections of their elementary education course to 69% among bachelors-level mathematics departments. Of the course coordinators, the majority were tenured or tenure-eligible, and in all types of departments, at least 90% of the coordinators were either tenured, tenure-eligible, or a full-time department member with a Ph.D.

TABLE SP.7 Among mathematics departments with multiple sections of their elementary mathematics education course, the percentage that administer their multiple sections in various ways, by type of department. Also, among departments with a course coordinator, the percentage with coordinators of various kinds, by type of department, in fall 2005.

	Mathematics Departments		
Departments with multiple sections of their Elementary Mathematics Education course	Univ (PhD)	Univ (MA)	College (BA)
Number with multiple sections	81	143	335
Percentage using same text for all sections	97%	91%	100%
Percentage with course coordinator	90%	82%	69%
Status of Course Coordinator			
a) Tenured/Tenure eligible	65%	81%	68%
b) Postdoc	0	0	0
c) Full-time visitor	2	9	0
d) Full-time, with Ph.D., not (a),(b),(c)	28	9	32
e) Full-time, without PhD, not (a),(b),(c)	2	0	0
f) Part-time	3	0	0
g) Graduate teaching assistant	0	0	0

TABLE SP.8 Percentage of mathematics departments estimating when K-8 pre-service teachers take their first mathematics education course, by type of department, in fall 2005.

	Mathematics Departments		
When Students Take K-8 Mathematics Education Course	Univ (PhD)	Univ (MA)	College (BA)
Freshman year	23%	43%	23%
Sophomore year	45	36	64
Junior year	27	17	13
Senior year	5	4	0

The final two tables in this part of Chapter 2 give data about other ways that departments participated in teacher education programs. Table SP.9 shows the number of departments of various types that offered *secondary* mathematics certification programs, and shows where students in those programs learned about the history of mathematics in fall 2005. Table SP.10 shows the extent to which mathematics and statistics departments were involved in graduate teacher education programs, either inside or outside of the department.

TABLE SP. 9 Number and percentage of mathematics departments in universities and four year colleges with secondary mathematics certification programs whose pre-service secondary teachers learn mathematics history in various ways, by type of department, in fall 2005.

	Mathematics Departments		
Mathematics Departments with Secondary Certification Programs	Univ (PhD)	Univ (MA)	College (BA)
Number	151	170	833
Percentage with a required mathematics history course for secondary certification	58%	69%	41%
Percentage with mathematics history only in other required courses for secondary certification	22	25	43
Percentage with no mathematics history requirement for secondary certification	19	7	16

TABLE SP.10 Degree of participation by mathematics and statistics departments in graduate mathematics education programs of various kinds, by type of department, in fall 2005.

	Mathematics Departments			Statistics Departments	
Participation in a Graduate Mathematics Education Program	Univ (PhD)	Univ (MA)	College (BA)	Univ (PhD)	Univ (MA)
Percentage with no graduate mathematics education courses	43	21	89	58	56
Percentage with mathematics education courses that are part of a degree program in their own department	29	35	2	23	29
Percentage with mathematics education courses that are part of a degree program in another department	28	44	9	19	15

Tables SP.11 to SP.15: Academic Resources Available to Undergraduates

In fall 2005, as in fall 2000, almost all two-year colleges reported using placement testing for incoming students. In CBMS2000, 67% of two-year colleges reported that their placement test led to mandatory placement. The CBMS2005 survey changed the question somewhat, and found that in fall 2005, 88% of public two-year colleges had mandatory placement based on the placement test or based on the placement test and other information. Table SP.11 also shows the source of placement tests used by public two-year colleges with placement testing programs. The use of locally written placement tests declined, falling from 99% of two-year colleges in fall 2000 to 11% in fall 2005. Because many two-year colleges indicated that they used placement tests from several sources, the percentages in Table SP.11 do not add to 100%.

TABLE SP.11 Percentage of public two-year colleges that have placement testing programs and use them in various ways, and the source of the placement tests, in fall 2005. (Data from fall 2000 in parentheses.)

	Percentage of two-year colleges %
That offer placement tests	97 (98)
That usually require placement tests of first-time enrollees	97 (98)
That require students to discuss placement scores with advisors	90 (79)
That use placement tests as part of mandatory placement	88 (na)
That periodically assess the effectiveness of their placement tests	81 (85)
Source of Placement Test	
Written by department	11 (99)
Provided by ETS	22 (30)
Provided by ACT	51 (34)
Provided by professional society	12 (3)
Provided by other external source	25 (26)

Table SP.12 shows that most mathematics departments in two-year colleges, and most mathematics and statistics departments in four-year colleges and universities, offered labs or tutoring centers for their students in fall 2005. The only major change since fall 2000 was the increase in the percentage of statistics departments that offered labs or tutoring centers (up from six out of ten to eight out of ten). Table SP.13 shows the types of assistance available in mathematics and statistics labs and tutoring centers. Among mathematics departments of four-year colleges and universities, the emphasis on computer use in the labs declined from the levels observed in fall 2000, while it increased in both statistics departments and two-year colleges. The use of para-professional and part-time faculty as tutors declined between 2000 and 2005, while tutoring by full-time faculty increased.

TABLE SP.12 Percentage of mathematics and statistics departments in four-year colleges and universities, and mathematics programs in public two-year colleges, that operate a lab or tutoring center in their discipline in fall 2005. (Fall 2000 data in parentheses)

Percentage with Lab or Tutoring Center	Mathematics Departments	Statistics Departments	Two-Year College Mathematics Programs
Univ (PhD)	96 (90)	79 (61)	--
Univ (MA)	91 (95)	85 (50)	--
Coll (BA)	88 (89)	--	--
All departments	89 (89)	80 (59)	95 (98)

TABLE SP.13 Among mathematics and statistics departments in four-year colleges and universities and mathematics programs in public two-year colleges that operate labs or tutoring centers, the percentage that offer various services, by type of department, in fall 2005. (Fall 2000 data in parentheses.)

Percentage Offering Various Services in Labs & Tutoring Centers	Computer-aided instruction %	Computer software %	Media such as video tapes %	Tutoring by students %	Tutoring by para-professional staff %	Tutoring by part-time faculty %	Tutoring by full-time faculty %	Internet resources %
Mathematics Departments								
Univ (PhD)	33	48	20	98	29	22	27	38
Univ (MA)	33	55	40	96	43	23	28	37
Coll (BA)	25	33	27	99	20	9	19	21
Total Mathematics Departments	27 (38)	38 (62)	27 (24)	98 (99)	24 (35)	13 (18)	21 (16)	25 (33)
Statistics Departments								
Univ (PhD)	44	68	13	96	13	9	17	27
Univ (MA)	51	83	17	100	17	0	17	69
Total Statistics Departments	46 (36)	71 (63)	14 (17)	97 (93)	14 (37)	7 (11)	17 (3)	37 (23)
Two-Year College Mathematics Programs	75 (68)	72 (69)	68 (74)	94 (96)	67 (68)	48 (48)	51 (42)	77 (53)

Note: 0 means less than one-half of 1%.

Tables SP.14 and SP.15 show the extent to which departments of various kinds made a spectrum of academic enrichment opportunities available to their undergraduates in fall 2005. These tables expand upon Table AR.12 in CBMS2000. With few exceptions, the percentage of departments offering a given academic opportunity increased between 2000 and 2005. Perhaps the most notable exception in Table SP.14 is the decline from 47% to 34% in the number of four-year mathematics departments that offer opportunities for their undergraduates to become involved with K–12 schools. The difference between mathematics and statistics departments in terms of the availability of the senior thesis option in fall 2005 (76% in mathematics departments, compared to 31% among statistics departments) may also be noteworthy.

TABLE SP.14 Percentage of mathematics programs at public two-year colleges, and of mathematics and statistics departments in four-year colleges and universities, that offer various kinds of special opportunities for undergraduates, by type of department, in fall 2005. (Fall 2000 data in parentheses.)

Percentage with Special Opportunities for Undergraduates	Honors sections of courses for majors %	Math or Stat club %	Special programs for women %	Special programs for minorities %	Math or Stat contests %	Special Math or Stat colloquia for undergrads %	Outreach in K–12 schools %
Mathematics Departments							
Univ (PhD)	70	88	15	10	92	70	51
Univ (MA)	44	92	21	23	68	71	63
Coll (BA)	18	66	4	6	62	37	26
Total Mathematics Depts	28 (20)	72 (61)	8 (9)	8 (7)	67 (63)	46 (54)	34 (47)
Statistics Departments							
Univ (PhD)	27	27	0	7	22	47	11
Univ (MA)	41	29	0	0	29	44	15
Total Statistics Depts	30 (46)	27 (25)	0 (2)	6 (2)	23 (28)	46 (41)	12 (7)
Two-Year College Mathematics Programs	24 (20)	22 (14)	7 (4)	15 (4)	37 (28)	6 (9)	25 (20)

Note: 0 means less than one-half of 1%.

TABLE SP.15 Percentage of mathematics programs in public two-year colleges, and of mathematics and statistics departments in four-year colleges and universities, that offer various additional special opportunities for undergraduates, by type of department, in fall 2005. (Fall 2000 data, where available, in parentheses.)

Percentage with Additional Opportunities for Undergraduates	Undergrad. research opportunity %	Indep. studies opportunity %	Assigned advisors in dept. %	Senior thesis opportunity %	Math career day %	Graduate school advising %	Internship opportunity %	Senior seminar opportunity %
Mathematics Departments								
Univ (PhD)	90	95	85	62	24	49	47	39
Univ (MA)	74	91	97	53	15	61	55	46
Coll (BA)	54	79	88	48	10	45	35	38
Total mathematics depts	62 (59)	83 (80)	89 (82)	50	12	47	39	39
Statistics Departments								
Univ (PhD)	60	62	73	27	15	56	47	15
Univ (MA)	59	100	85	44	15	59	71	29
Total statistics depts	60 (58)	70 (67)	76 (71)	31	15	57	52	18
Two-Year College Mathematics Programs	9 (4)	38 (25)	40 (33)	na	na	na	na	na

Tables SP.16 and SP.17: Dual Enrollments—College Credit for High School Courses

Dual-enrollment courses are courses taught in high school by high school instructors for which high school students receive both high school and college credit. This arrangement is *not* the same as obtaining college credit based on AP or IB examination scores. Dual enrollment is encouraged by many state governments as a way to utilize state-wide educational resources more efficiently.

In fall 2000, most dual-enrollment courses involved an agreement between a high school, where the course was taught, and a local two-year college that awarded college credit for the course. In many states, public four-year colleges and universities were required to count such dual-enrollment credits toward their graduation requirements. Based on CBMS2000 findings, the Mathematical Association of America Board of Governors called for careful tracking of dual-enrollment growth and related quality-control issues, and CBMS2005 agreed to study dual-enrollment issues in fall 2005 in both two- and four-year colleges and universities.

Table SP.16 shows that dual-enrollment courses were widespread among two-year colleges in fall 2005, with about 50% of all public two-year colleges awarding college credit for some dual-enrollment courses. In fall 2005 there were about 58,000 enrollments in Precalculus at two-year colleges, and about 14,000 dual-enrollments in high school versions of that same course, meaning that just over 19% of all credit in Precalculus awarded by two-year colleges was earned in dual-enrollment courses. Also, there were about 51,000 enrollments in Calculus I courses taught in two-year colleges, and about 11,000 enrollments in the dual-enrollment version of that same course. Consequently, about 18% of all Calculus I credit awarded by two-year colleges was through dual enrollments.

Comparing enrollment percentages for fall 2005 with data from CBMS2000 is somewhat problematic because the CBMS2000 survey asked two-year colleges to report the number of dual-enrollment sections rather than the number of dual enrollments. Nevertheless, it may be worth noting that CBMS2000 found that in fall 2000, about 18% of two-year college sections in Precalculus and about 15% of two-year college Calculus I sections were dual-enrollment sections.

In fall 2000, anecdotal evidence suggested that few of the nation's four-year colleges and universities were involved in granting dual-enrollment credit for high school mathematics and statistics courses, so that four-year departments were not asked to report on their dual-enrollment activity. Table SP.16 of CBMS2005 shows that in fall 2005, about one in seven mathematics departments, and one in twelve statistics departments, at four-year colleges and universities had entered into dual-enrollment agreements with high schools. However, in fall 2005 the number of dual-enrollment registrations in four-year colleges and universities was small compared to the number of traditional enrollments. For example, the number of dual enrollments in College Algebra and in Calculus I were only about 4% of the number of regular enrollments in those courses. In statistics departments, the number of dual enrollments in Elementary Statistics was about 3% of traditional enrollments in that same course.

A major concern in dual-enrollment courses is the degree of quality control exercised by the two-year or four-year department through which college-level credit for the courses is awarded. Table SP.16 examines several types of quality control that college-level departments might have had over their dual-enrollment courses in fall 2005, and presents comparison data for dual-enrollment programs of two-year colleges from fall 2000. (Comparable data from fall 2000 do not exist for dual-enrollment programs at four-year colleges and universities.) CBMS2000 showed that in fall 2000, 79% of two-year colleges reported that they always controlled the choice of the textbook used in their dual-enrollment courses. By the fall of 2005, that percentage dropped slightly, to 74%, and the corresponding percentage of "never control the textbook" responses grew from 10% in fall 2000 to 14% in fall 2005. Both final exam design and the choice of instructor in dual-enrollment courses seemed to drift away from two-year colleges' control between 2000 and 2005, with the largest change occurring in the degree of control over the final examination. Only in the area of syllabus design or approval did the degree of control by two-year colleges in dual-enrollment courses seem to increase between fall 2000 and fall 2005. Four-year college and university mathematics departments that were involved in dual-enrollment programs in fall 2005 exercised a degree of course control roughly similar to that of two-year college mathematics programs, except in terms of the choice of textbook, an area in which four-year departments had considerably less control than two-year departments.

Monitoring teaching quality is another opportunity for quality-control in dual-enrollment courses. About two-thirds of two-year colleges monitored the teaching of dual-enrollment instructors, while among four-year mathematics departments the number was closer to one in six. The findings reported in Table SP.16 will not be reassuring to those who expect two- and four-year colleges and universities to control the content and depth of courses for which they are granting college credit.

TABLE SP.16 Percentage of departments offering dual-enrollment courses taught in high school by high school teachers, enrollments in various dual-enrollment courses in spring 2005 and fall 2005, compared to total of all other enrollments in fall 2005, and (among departments with dual enrollment programs) percentage of various departmental controls over dual-enrollment courses, by type of department. (Fall 2000 data in parentheses.)

	Four-year Mathematics			Two-year Mathematics			Four-year Statistics		
Percentage of Departments with Dual-Enrollment Courses	14%			50%			8%		
Number of Dual Enrollments	Dual enrollments spring 2005	Dual enrollments fall 2005	Other enrollments fall 2005	Dual enrollments spring 2005	Dual enrollments fall 2005	Other enrollments fall 2005	Dual enrollments spring 2005	Dual enrollments fall 2005	Other enrollments fall 2005
College algebra	2673	8046	201000	9913	11362	206000	na	na	na
Precalculus	2944	597	93000	14650	13801	58000	na	na	na
Calculus I	5540	8490	201000	8218	11188	51000	na	na	na
Statistics	340	981	124000	3648	2440	111000	1563	1295	43000
Other	3470	723	na	5452	3045	na	0	0	na
Dept. Control of Dual Enroll. Courses Taught by H S Teachers	Never	Sometimes	Always	Never	Sometimes	Always	Never	Sometimes	Always
Textbook choice	41%	15%	44%	14% (10)	12% (12)	74% (79)	36%	30%	34%
Syllabus design/approval	2%	6%	92%	4% (8)	7% (11)	89% (82)	36%	0%	64%
Final exam design	40%	30%	30%	36% (15)	28% (28)	37% (57)	100%	0%	0%
Choice of instructor	32%	20%	48%	35% (19)	13% (20)	52% (61)	36%	0%	64%
Departmental teaching evaluations required in dual enrollment courses			16%			64% (67)			0%

Table SP.17 describes a relatively new phenomenon, in which colleges and universities send their own faculty members out into high schools to teach courses that grant both high school and college credit. About one in twenty-five mathematics departments in four-year colleges and universities had such programs in fall 2005, as did about one in eight public two-year colleges. The number of students involved in these programs was small compared to the number of dual-enrollment students taught by high school teachers.

TABLE SP. 17 Percentage of departments in four-year colleges and universities and in public two-year colleges that assign their own full-time or part-time faculty members to teach courses in a high school that award both high school and college credit, and number of students enrolled, in fall 2005.

	Four-year Mathematics Departments	Two-year Mathematics Departments	Statistics Departments
Assign their own members to teach dual-enrollment courses	4%	12%	0%
Number of students enrolled	2874	2008	0

Table SP.18: Mathematical Sciences and General Education Requirements

Table SP.18 examines the role of mathematics and statistics courses in the general education requirements of U.S. colleges and universities in fall 2005. Because of the wide variety of academic structures in U.S. universities, CBMS2005 began by asking each department whether its own academic unit had a quantitative requirement for bachelors degrees granted through that academic unit. The phrase "its own academic unit" was designed to address a situation, widespread in universities, in which a mathematics department belonged to a college (say the Arts and Sciences College), and all students of that college were required to take a quantitative course of some kind, even though students in some of the university's other colleges (say the College of Fine Arts) did not need to do so.

Table SP.18 shows that in almost nine out of ten cases, the academic unit to which the four-year mathematics and statistics departments belonged did have a quantitative requirement in fall 2005. In a majority of those cases, the mathematics department reported that the only way for a student to fulfill the quantitative requirement was by taking a course in the mathematics department. About one-quarter of the time, any mathematics course was adequate to fulfill the requirement, and in the other cases only certain mathematics courses fulfilled the requirement. Asked which departmental courses could satisfy general education requirements, departments most frequently mentioned Calculus, followed closely by Elementary Statistics, College Algebra, Precalculus, and a special general education course in the department. Among the several freshman mathematics course options proposed in the CBMS2005 questionnaire, all but one seemed to satisfy general education requirements in a majority of mathematics departments, the exception being "a mathematical models course." In statistics departments, the elementary statistics course was the primary general education course in the department.

TABLE SP.18: Percentage of four-year mathematics and statistics departments whose academic units have various general education requirements, and the department's role in general education, by type of department in fall 2005.

	Four-year Mathematics Departments			Statistics Departments	
General Education	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %
There is a quantitative requirement in the department's college	87	98	91	86	88
The quantitative requirement must be taken in the department	51	68	61	8	0
Any freshman course in the department fulfills the quantitative requirement	26	28	32	27	17
Only certain departmental courses fulfill quantitative requirement	74	72	69	73	83
Departmental courses satisfying the quantitative requirement					
College algebra or Precalculus	56	61	62	na	na
Calculus	97	87	86	na	na
Mathematical models	23	11	13	na	na
A probability/statistics course	55	60	66	94	60
Statistical literacy	na	na	na	27	20
A special general education course in the department	52	73	55	0	0
Some other course(s) in the department	50	71	57	33	20

Tables SP.19 to SP.25: Curricular Requirements of Mathematics and Statistics Majors in the U.S.

In the CBMS2000 report, Table SE.5 presented data on the percentage of mathematics and statistics departments that offered certain upper-division courses in the 2000–2001 academic year. Based on course availability, CBMS2000 concluded that in fall 2000, there were large differences between the kind of mathematical sciences major available to students in doctoral-level departments and in bachelors-level departments. In response to a request from the MAA Committee on the Undergraduate Program in Mathematics, CBMS2005 collected data about specific requirements of majors, about course-offering patterns for all upper-division mathematics and statistics courses during the two-year window consisting of the 2004–2005 and 2005–2006 academic years, and about the extent to which a student could use interdisciplinary components from another mathematical science (e.g., upper-division courses in statistics and computer science) to fulfill the requirements of a mathematics major.

Obtaining national data on the requirements of the mathematics major in fall 2005 was complicated because most mathematics departments offer several different tracks within the mathematics major, each with its own set of requirements. For example, there might be an applied mathematics track, another track for students intending to teach mathematics in high school, another track that focuses on probability and statistics, another designed for students planning for mathematics graduate school, etc., etc. (Some departments refer to these tracks as being separate majors, but in this report we will refer to them as separate tracks within the departmental major.)

In fall 2005, was there any course seen as so central to mathematics that it was required in all of a department's potentially many tracks? Table SP.19 shows that a computer science course comes closest of all to being a universal requirement for U.S. mathematics majors. Real Analysis I, Modern Algebra I, and a statistics course were essentially tied for second place, with about a third of departments reporting that these courses were required in each track of their majors. Capstone experiences (e.g., senior project, thesis, seminar, internship) were widespread requirements in masters- and bachelors-level departments, but not in doctoral departments.

Long ago, many mathematics majors required two semesters of analysis and two semesters of modern algebra. CBMS2005 asked departments whether all, some, or none of the tracks within their major required Modern Algebra I plus another upper-division algebra course, and posed an analogous question about Real Analysis I plus another upper-division analysis course. A large majority of departments reported that in fall 2005, none of the tracks within their majors required two semesters of modern algebra courses, and that none of the tracks within their majors required two semesters of upper-division analysis courses. More specifically, at least seven out of ten bachelors departments reported that none of their tracks required two semesters of analysis, and that none of their tracks required two semesters of algebra. Even among doctoral departments, the majority reported that no track within the department required two semesters of algebra.

TABLE SP.19: Percentage of four-year mathematics departments requiring certain courses in all, some, or none of their majors, by type of department, in fall 2005.

Mathematics Department Requirements	Required in all majors			Required in some but not all majors			Not required in any major		
	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %	College (BA) %
Modern Algebra I	24	48	56	59	42	36	18	10	8
Modern Algebra I plus another upper division algebra course	5	8	8	40	28	17	55	63	75
Real Analysis I	36	39	46	49	54	29	15	7	25
Real Analysis I plus some other upper division analysis course	10	4	8	49	36	20	41	60	71
At least one computer science course	55	76	64	27	16	14	18	8	22
At least one statistics course	32	56	32	40	32	32	28	11	35
At least one upper division applied mathematics course	16	23	21	52	41	25	32	36	54
A capstone experience (senior project, thesis, seminar, internship)	27	52	59	23	13	8	50	35	33
An exit exam (written or oral)	8	8	29	4	16	3	88	76	68

Table SP.20 shows that in fall 2005, at least three-quarters of all doctoral statistics departments required three semesters of calculus, including multi-variable calculus, plus Linear Algebra, for all tracks of their majors. At the other end of the spectrum, almost two-thirds of all statistics departments reported that they do not require any applied mathematics course (beyond calculus courses and Linear Algebra) in any track of their majors.

TABLE SP.20 Percentage of statistics departments requiring certain courses in all, some, or none of their majors, by type of department, in fall 2005.

	Required in all majors		Required in some but not all majors		Not required in any major	
Percentage of Statistics Departments that Require	Univ (PhD) %	Univ (MA) %	Univ (PhD) %	Univ (MA) %	Univ (PhD) %	Univ (MA) %
(a) Calculus I	92	86	4	0	4	14
(b) Calculus II	87	86	4	0	8	14
(c) Multivariable Calculus	78	51	9	17	13	31
(d) Linear algebra/Matrix theory	84	69	3	0	13	31
(e) at least one Computer Science course	72	86	16	0	12	14
(f) at least one applied mathematics course, not incl. (a), (b), (c), (d)	24	14	12	17	64	69
(g) a capstone experience (e.g., a senior thesis or project, seminar, or internship)	34	51	9	17	57	31
(h) an exit exam(oral or written)	0	0	0	17	100	83

In fall 2005, to what extent did the nation's mathematics majors include interdisciplinary linkages with computer science and statistics? As noted above, an introductory computer science course was perhaps the most universal course requirement for a mathematics major. But were any upper-division courses in computer science allowed to count toward a track within the mathematics department major? If CBMS2005 data are interpreted conservatively, some answers are possible. For example, Table SP.21 shows that 69% of all doctoral mathematics departments allow some upper-division computer science course from another department to count toward one of their mathematics major tracks. In addition, 17% of doctoral mathematics departments teach upper-division computer science courses themselves, and it is reasonable to suppose that some mathematics major tracks in such departments might include some of the department's own upper-level computer science courses. Therefore, between 69% and 86% of doctoral mathematics departments allow upper-division computer science courses to count toward the requirements of some of their mathematics major tracks, while at least 14% do not allow any upper-division computer science courses to fulfill requirements of their majors. Table SP.21 shows that between 42% and 64% of bachelors-level mathematics departments allow upper-level computer science courses to count toward their requirements for some tracks, leaving at least 36% that do not.

The percentages in Table SP.21 suggest that in fall 2005, a large majority of mathematics departments allowed upper-level statistics courses (either from their own department or from another department) to count toward the requirements of one of their majors.

Table SP.21 shows that among doctoral statistics departments, 55% allowed upper-level computer science courses from other departments to count towards a track within the statistics major, and four percent taught upper-level computer science courses of their own. Consequently, about 40% of doctoral statistics departments did not allow any upper-division computer science courses to count toward their departmental statistics major. Table SP.21 also shows that two out of three doctoral statistics departments allowed some upper-division mathematics courses to count toward the requirements of some statistics major track.

TABLE SP.21 Percentage of mathematics departments and statistics departments that allow upper division courses from other departments to count toward their undergraduate major requirements, by type of department, in fall 2005.

	Four-year Mathematics Departments			Statistics Departments	
Percentage of Departments that	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %
Teach upper level computer science	17	25	42	4	29
Allow upper level CS courses from other depts. to count toward their major	69	31	22	55	100
Teach upper level statistics	64	94	87	na	na
Allow upper level statistics courses from other depts. to count toward their major	55	12	15	na	na
Allow upper division mathematics courses to count toward their major	na	na	na	66	86

Table SP.22 examines the availability of many upper-division courses in mathematics departments during the two-year window consisting of the consecutive academic years 2004–2005 and 2005–2006 (which we abbreviate as 2004–2005–2006). Analogous data for a smaller course list during the single academic years 1995–1996 and 2000–2001 appears in Table SE.5 of the CBMS2000 report. All other things being equal, one would expect to see a larger percentage of departments offering a given course during a two-year window than during a one-year window, and in most cases that is what Table SP.22 shows.

It is somewhat surprising that only about 61% of all four-year college and university mathematics departments offered Modern Algebra during the two-year window 2004–2005–2006, compared to a 71% figure for mathematics departments offering the same course during the single academic year 2000–2001 and a 77% figure for Modern Algebra in the single academic year 1995–1996. Similarly surprising is the percentage of all mathematics departments that offered a course called Real Analysis/Advanced Calculus: 70% for the 1995–1996 academic year, 56% for the 2000–2001 academic year, and 66% for the two-academic-year window 2004–2005–2006. These percentages, combined with the course-requirement data in Table SP.19, suggest that Modern Algebra and Real Analysis no longer hold the central position in the undergraduate mathematics major that they once did.

It may be worth noting that the percentage of bachelors-level mathematics departments offering Number Theory and Combinatorics was larger in 2004–2005–2006 than in 2000–2001, but the importance of this observation is tempered by the fact that less than a third of bachelors-level departments offered these courses in 2004–2005–2006.

Table SP.22 reinforces the tentative conclusion from CBMS2000 that there was a real difference between the mathematics major available to students in doctoral departments and in bachelors departments. For example, during the academic year 2000–2001, 87% of doctoral mathematics departments offered a Modern Algebra course, compared to 63% of bachelors departments. During the two-year window 2004–2005–2006, 86% of doctoral mathematics departments offered a Modern Algebra course, compared to 52% of bachelors-level departments. The situation for Real Analysis is similar: in 2000–2001, about 90% of doctoral mathematics departments offered Real Analysis, compared to 45% of bachelors-level departments, and during the two-year window 2004–2005–2006, 95% of doctoral departments and 57% of bachelors departments offered the course. The course-availability gaps between doctoral and bachelors departments for Geometry and Number Theory were larger, and specialized courses such as Combinatorics and Logic/Foundations were four times as likely to be available in doctoral mathematics departments than in bachelors-level departments.

Table SP.23 examines the analogous question for upper-level statistics courses taught in mathematics or in statistics departments. Among mathematics departments, for example, the percentage offering Mathematical Statistics in the two-year window 2004–2005–2006 was 38%, compared to a figure of 52% for the same course during the single academic year 2000–2001. The percentage of statistics departments that offered Mathematical Statistics in 2000–2001 was 90% and dropped to 76% in the two-year window 2004–2005–2006. Indeed, of the thirteen upper-division statistics courses in Table SP.23, ten were offered less frequently in statistics departments during the two-year window 2004–2005–2006 than during the one-year window 2000–2001. The exceptions were probability courses, biostatistics courses, and statistics senior seminars.

Tables SP.22 and SP.23 provide availability data for a broad spectrum of upper-division mathematics and statistics courses and could serve as baseline data for a future study of the evolution of the national mathematics and statistics curriculum between 2004–2005–2006 and 2009–2010–2011.

TABLE SP.22 Percentage of mathematics departments offering various upper-division mathematics courses at least once in the two academic years 2004-2005 and 2005-2006, plus historical data on the one year period 2000-2001, by type of department.

		Academic Years 2004-2005 & 2005-2006			
	All Math Depts 2000-01 %	All Math Depts 2004-5 & 2005-6 %	PhD Math %	MA Math %	BA Math %
Upper-level Mathematics Courses					
Modern Algebra I	71	61	86	87	52
Modern Algebra II	na	21	40	40	15
Number Theory	33	37	61	61	29
Combinatorics	18	22	55	38	14
Actuarial Mathematics	na	11	24	23	6
Foundations/Logic	16	11	27	16	7
Discrete Structures	na	14	27	22	10
History of Mathematics	na	35	43	68	28
Geometry	56	55	81	89	44
Math for secondary teachers	42	37	41	50	35
Adv Calculus/ Real Analysis I	56	66	95	86	57
Adv Calculus/Real Analysis II	na	26	62	44	17
Adv Mathematics for Engineering/Physics	na	16	50	28	7
Advanced Linear Algebra	na	19	52	42	9

TABLE SP.22, continued

Upper-level Math, Continued	All Math Depts 2000-01 %	Academic Years 2004-2005 & 2005-2006: All Math Depts 2004-5 & 2005-6 %	PhD Math %	MA Math %	BA Math %
Vector Analysis	na	9	21	6	7
Advanced Differential Equations	na	13	45	28	5
Partial Differential Equations	na	19	57	29	11
Numerical Analysis I and II	na	47	83	76	36
Applied Math/Modeling	24	26	48	47	18
Complex Variables	na	37	80	53	26
Topology	22	32	61	33	26
Mathematics of Finance	na	8	24	8	5
Codes & Cryptology	na	8	17	8	7
Biomathematics	na	8	24	9	4
Intro to Operations Research	13	12	17	20	10
Intro to Linear Programming	na	6	19	21	1
Math senior seminar/Ind study	58	45	61	48	42

TABLE SP.23 Percentage of mathematics and statistics departments offering various undergraduate statistics courses at least once in academic year 2000-2001 and at least once in the two academic years 2004-2005 and 2005-2006, by type of department.

Upper Level Statistics Courses	All Math Depts 2000-01 %	AY 2004-05 & 2005-06: All Math Depts %	AY 2004-05 & 2005-06: PhD Math %	AY 2004-05 & 2005-06: MA Math %	AY 2004-05 & 2005-06: BA Math %	All Stat Depts 2000-01 %	AY 2004-05 & 2005-06: All Stat Depts %	AY 2004-05 & 2005-06: PhD Stat %	AY 2004-05 & 2005-06: MA Stat %
Mathematical Statistics	52	38	52	63	31	90	76	73	88
Probability	40	51	72	69	43	75	86	90	73
Stochastic Processes	6	6	21	13	2	46	43	42	44
Applied Statistical Analysis	13	13	26	32	7	72	65	63	73
Experimental Design	10	6	14	23	2	74	54	49	73
Regression & Correlation	9	6	20	12	3	82	62	55	88
Biostatistics	5	4	11	13	2	20	25	28	15
Nonparametric Statistics	4	2	6	8	0	45	38	33	59
Categorical Data Analysis	1	1	5	3	1	39	21	19	29
Sample Survey Design	3	4	13	8	1	52	49	43	73
Stat Software & Computing	5	3	11	7	1	48	43	35	73
Data Management	1	0	0	0	0	13	5	6	0
Statistics Senior Sem/Ind Study	5	3	8	8	1	34	41	36	59

Note: 0 means less than one-half of one percent.

Table SP.24 summarizes responses from mathematics and statistics departments about the career plans of their bachelors graduates from the 2004–2005 academic year. Departments were asked to give their best estimates of the percentages of their graduates who chose this or that post-college path; the question did not ask departments to do follow-up studies of the previous year's graduates. Consequently, the first four rows should be taken with a grain of salt, and the table does not answer the question "What did mathematics majors (statistics majors) do after graduation?" But it may say something about the extent to which mathematics and statistics departments know their graduating seniors.

TABLE SP.24 Departmental estimates of the percentage of graduating mathematics or statistics majors from academic year 2004-2005 who had various post-graduation plans, by type of department in fall 2005.

	Mathematics Departments			Statistics Departments	
Departmental Estimates of Post-college Plans	Univ (PhD)	Univ (MA)	College (BA)	Univ (PhD)	Univ (MA)
Students who went into pre-college teaching	16%	44%	32%	1%	0%
Students who went to graduate or professional school	21	16	19	18	29
Students who took jobs in business, government, etc.	19	21	29	16	36
Students who had other plans known to the department	4	1	2	0	6
Students whose plans are not known to the department	39	18	17	65	28

Table SP.25: Assessment Activities in Mathematics and Statistics Departments.

During the ten-year period leading up to 2005, state governments, national accrediting agencies, and professional organizations such as the Mathematical Association of America all placed great emphasis on departmental assessment studies [MAAGuidelines], [M], [CUPM], [GKM]. For further information, see http://www.maa.org/saum/index.html.

Table SP.25 summarizes departmental responses about their assessment activities during the period 1999–2005. Surveying departmental graduates was the most widely used assessment technique among masters- and bachelors-level mathematics departments and was also used by six out of ten doctoral mathematics departments. Other recommended assessment techniques were less widely used. Less than half of all mathematics departments used outside reviewers as part of their assessment efforts, perhaps because of cost issues. Less than half of all departments consulted “client departments,” i.e., departments whose courses use mathematics or statistics courses as prerequisites, to see whether the client departments were satisfied with what their students had learned in mathematics courses. Less than half of all departments did follow-up studies to determine how well the department’s courses prepared the department’s own students for later departmental courses. But whatever assessment techniques were or were not used, Table SP.25 reports that in three quarters of mathematics departments, assessment efforts led departments to change their undergraduate programs.

TABLE SP.25 Percentage of four-year mathematics and statistics departments undertaking various assessment activities during the last six years, by type of department, in fall 2005.

	Four-year Mathematics Departments			Statistics Departments	
Percentage Using Various Assessment Tools	Univ (PhD) %	Univ (MA) %	College (BA) %	Univ (PhD) %	Univ (MA) %
Consult outside reviewers	47	45	29	37	59
Survey program graduates	62	81	74	54	71
Consult other departments	51	41	35	29	56
Study data on students' progress in later courses	45	52	38	30	56
Evaluate placement system	72	72	51	5	15
Change undergraduate program due to assessment	76	72	76	69	29